



## Voltage Stability Improvement on Optimal placement of FACTS Devices

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### ABSTRACT

Voltage stability challenge in power systems remains one of the major concerns in system planning and operation. The ability of a power system to maintain acceptable voltage levels at all buses in the system under normal operating conditions and during contingencies pose a major headache for power system researchers and practitioners. As power systems become more complex, coupled with environmental concerns, land scarcity and huge capital requirements in doing of new power lines and substations, voltage instability becomes an increasingly serious challenge that requires out of the box solutions. FACTS devices is a group of highly flexible and versatile controllers that regulate active and reactive power flows in real time to enhance system controllability and increase the power transfer capability. The UPFC is one of the most complex and versatile FACTS devices that shall be used in the research.

Proper knowledge of how close the actual system's operating voltages are from the voltage stability limits is crucial for the optimal placement of FACTS devices. This research uses static methods, namely the Voltage Stability indices for optimal placement of the FACTS devices for security-constrained voltage stability improvement. The research shall be done by the use of Power System Analysis Toolbox (PSAT) software that runs on MATLAB's environment on the IEEE 39-Bus 10-Generator test system.

**Key words:** FACTS, Optimal, Security-Constrained, UPFC and Versatile

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### INTRODUCTION

The demand for clean, reliable and affordable power is growing at unprecedented rate due to rapid industrialization in many countries. Rapid growth in power generation, transmission and distribution has come along with increased power supply quality challenges among them, the challenge of voltage stability. Voltage stability is the ability of a power system to maintain acceptable voltage levels under normal operating conditions and after being subjected to disturbances such as a sudden increase in load or loss of generation. Conventional voltage stability improvement methods such as capacitor banks, reactors and transformers can be used to provide steady state voltage control. However, these devices are based on electro-mechanical control among other drawbacks thus impeding high speed and real-time control. This in essence means that they lack the much sought after traits of operational flexibility, versatility and real-time control [1-3]. Due to their inherent advantages over the conventional voltage control methods, FACTS-Flexible Alternating Current Transmission System- devices have been increasingly used as an alternative over the years. Research on the location of the FACTS devices using such methods as small signal analysis, hopf bifurcation, time domain analysis, loss sensitivity factors, fuzzy index and voltage change index has been well documented [4-7].

Static and dynamic methods can be used for voltage stability studies. Dynamic methods apply real-time simulation in time domain using precise dynamic models. Static methods solve specific first or second order functions or indices derived from the power flow equations of the network which show the capability of the power system to remain stable. They run with specific load increases until the voltage collapse point is reached thus allowing the examination of a wide range of system operating conditions such as heavy loading and contingencies. The objective

of this study is to use the voltage stability indices in predicting the proximity to voltage collapse as one of the static methods for the optimal location of the FACTS devices [8-10]. Two versatile voltage stability indices, namely the Line stability Index and the Fast Voltage Stability Index shall be used in this research to do the optimal placement of FACTS devices. The research shall also take care of the objective of system voltage security thus the use of voltage security-constrained load flow analysis as the base case. An analysis of the System's voltage profiles before and after the installation of FACTS devices shall be done.

The paper starts with a general introduction followed by the Mathematical Modelling of the Unified Power Flow Controller and the Voltage Stability indices, the methodology used, the results obtained, discussion of the same and finally a conclusion.

**UNIFIED POWER FLOW CONTROLLER (UPFC)**

The basic operating principle of an UPFC is as shown in Fig. 1. The UPFC consists of two switching converters based on Voltage Source Converter valves connected by a common DC link [11-17]. Based on Fig.1 above an equivalent circuit as shown in Fig.2 below can be established.

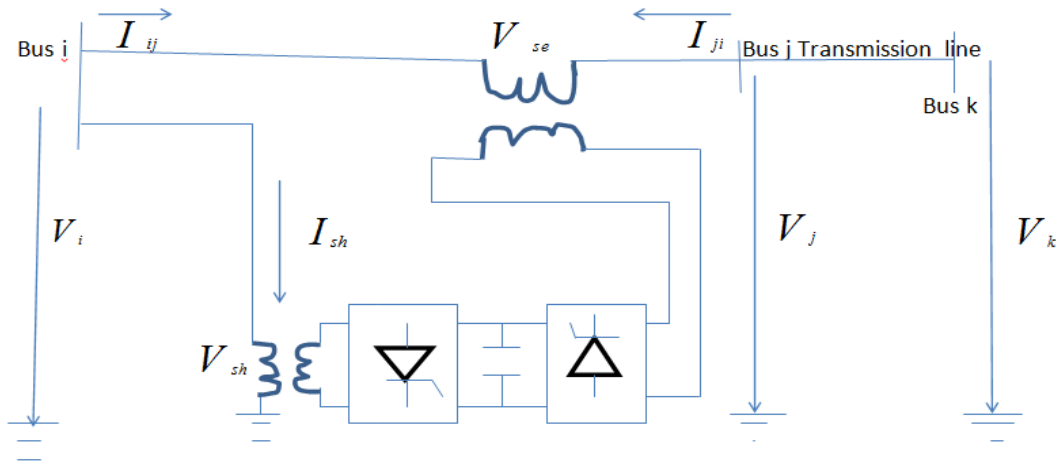


Fig.1 Operating Principle of UPFC

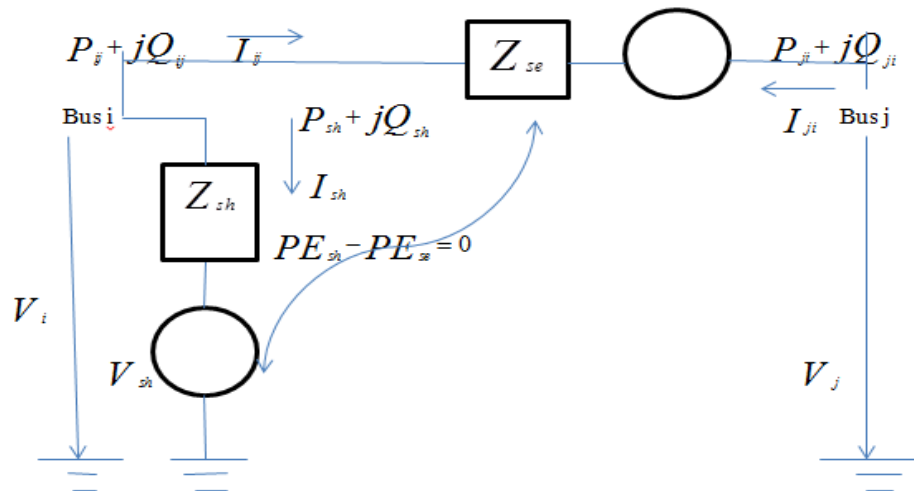


Fig.2 Equivalent circuit of UPFC

In Fig. 2, the phasors  $V_{sh}$  and  $V_{se}$  represent the equivalent injected shunt and series voltages respectively and  $Z_{sh}$  and  $Z_{se}$  are the UPFC series and shunt coupling transformer impedances respectively.  $V_i$  and  $V_j$  are the voltages at buses  $i, j$  respectively while  $V_k$  is the voltage of bus  $k$  of the receiving-end of the line.  $I_{sh}$  is the current through the UPFC shunt converter.  $P_{sh}$  and  $Q_{sh}$  are the shunt converter branch active and reactive power flows respectively.  $I_{ij}$  and  $I_{ji}$  are the currents through the UPFC series converter.  $P_{ij}$  and  $Q_{ij}$  are the UPFC series active and reactive power flows respectively leaving bus  $i$ .  $P_{se}$  is the real power exchange of the series converter with the DC link.

For the equivalent circuit of UPFC in Fig.2, if

$$V_{sh} = V_{sh} < \theta_{sh}, V_{se} = V_{se} < \theta_{se}, V_i = V_i < \theta_i, V_j = V_j < \theta_j,$$

then the power flow constraints of the UPFC shunt and series branches are:

$$P_{sh} = V_i^2 g_{sh} - V_i V_{sh} \{g_{sh} \cos(\theta_i - \theta_{sh}) + b_{sh} \sin(\theta_i - \theta_{sh})\} \quad (1)$$

$$Q_{sh} = -V_i^2 b_{sh} - V_i V_{sh} \{g_{sh} \sin(\theta_i - \theta_{sh}) - b_{sh} \cos(\theta_i - \theta_{sh})\} \quad (2)$$

$$P_{ij} = V_i^2 g_{ij} - V_i V_j \{g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}\} - V_i V_{se} \{g_{ij} \cos(\theta_i - \theta_{se}) + b_{ij} \sin(\theta_i - \theta_{se})\} \quad (3)$$

$$Q_{ij} = -V_i^2 b_{ij} - V_i V_j \{g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}\} - V_i V_{se} \{g_{ij} \sin(\theta_i - \theta_{se}) - b_{ij} \cos(\theta_i - \theta_{se})\} \quad (4)$$

$$P_{ji} = V_j^2 g_{ij} - V_i V_j \{g_{ij} \cos \theta_{ji} + b_{ij} \sin \theta_{ji}\} + V_j V_{se} \{g_{ij} \cos(\theta_j - \theta_{se}) + b_{ij} \sin(\theta_j - \theta_{se})\} \quad (5)$$

$$Q_{ji} = -V_j^2 b_{ij} - V_i V_j \{g_{ij} \sin \theta_{ji} - b_{ij} \cos \theta_{ji}\} + V_j V_{se} \{g_{ij} \sin(\theta_j - \theta_{se}) - b_{ij} \cos(\theta_j - \theta_{se})\} \quad (6)$$

$$\text{where } g_{sh} + jb_{sh} = 1/Z_{sh}, g_{ij} + jb_{ij} = 1/Z_{se}, \theta_{ij} = \theta_i - \theta_j, \theta_{ji} = \theta_j - \theta_i$$

### VOLTAGE STABILITY INDICES

The Voltage Stability Indices are generated from the load flow equations. The indices show the system's stability condition and can be used to estimate the systems operating states [18]. The mathematical expression of a Voltage Stability Index (VSI) is written as a polynomial containing the system's real-time measurements such as voltage magnitudes, phase angles, bus injected power and branch power flow values. The values of VSI are distinctly different in normal condition and contingencies for a power system. The changing of the VSI values from no load condition to maximum permissible loading condition reflects the system's stability trend from stable to unstable. The point when the system loses stability is called the optimal point.

Voltage magnitude is the most often used parameter in voltage stability index studies. A typical Voltage Stability Analysis considering voltage magnitude is based on a simplified two-bus Thevenin Equivalent power system with line resistance neglected. The approximate power flow equations through sending and receiving ends are obtained. One VSI method considering voltage magnitude of the receiving end is derived. The method utilizes the approximation of neglecting the line resistance for transmission lines with a high reluctance/resistance ratio. The approximated maximum active/reactive and apparent power flow values are obtained by using power flow measurements to express the voltage magnitude at receiving end and calculating its minimum value.

The Fast Voltage Stability Index – FVSI-is an indicator based on measurements of voltages and reactive power. It is a very good indicator of the weakest lines in the network for mitigation such as placement of FACTS devices [19-24]. The line model used to derive the indicator is shown in Fig. 3.

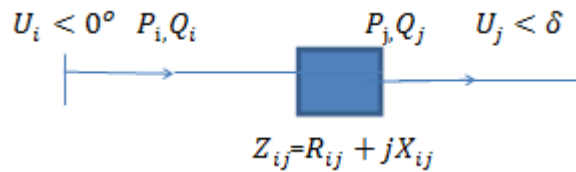


Fig.3 2-Bus Line Model for derivation of Voltage Stability Indices

We have two Buses namely Bus i (sending) and Bus j (receiving).  $U_i$  is the sending end voltage,  $Q_j$  the reactive power at the receiving end. Bus i is used as the reference bus, with the voltage angle set to 0.

The derivation of the index begins with the general equation for the current in a line between two Buses i and j as:

$$I_{ij} = \frac{U_i - U_j}{Z_{ij}} \quad (7)$$

The apparent power received at Bus j is found by multiplying 1 with the voltage at Bus j as:

$$I_{ij} = U_j \frac{U_i - U_j}{Z_{ij}} = P_j + jQ_j \quad (8)$$

The imaginary part of 2 is the reactive power received at Bus j given by equation:

$$Q_j = \frac{U_i U_j (R_{ij} \sin \delta + X_{ij} \cos \delta) - X_{ij} U_j^2}{R_{ij}^2 + X_{ij}^2} \quad (9)$$

This can be rewritten as a second-order equation for  $U_j$  as:

$$U_j^2 - U_j U_i \left( \frac{R_{ij}}{X_{ij}} \sin \delta + \cos \delta \right) + \left( X_{ij} + \frac{R_{ij}^2}{X_{ij}} \right) Q_j = 0 \quad (10)$$

FVSI is based on the principle that the system remains stable as long as there are only real solutions to equation 4 above.

$$\text{i.e.} \quad \left[ \left( \frac{R_{ij}}{X_{ij}} \sin \delta + \cos \delta \right) U_i \right]^2 - 4 \left( X_{ij} + \frac{R_{ij}^2}{X_{ij}} \right) Q_j \geq 0 \quad (11)$$

Simplifying 5 above and assuming that the angle difference  $\delta$  is normally very small ( $\delta \approx 0, R_{ij} \sin \delta \approx 0$  and  $X_{ij} \cos \delta \approx X_{ij}$ ) gives:

$$(X_{ij} U_i)^2 \geq 4 \left( X_{ij} + \frac{R_{ij}^2}{X_{ij}} \right) Q_j \quad (12)$$

FVSI is thus defined as the ratio between the two terms:

$$FVSI = \frac{4 Z_{ij}^2 Q_j}{U_i^2 X_{ij}} \leq 1 \quad (13)$$

As shown in equation 7, the power transmission through line i-j is stable as long as  $FVSI \leq 1$

Line stability Index  $L_{mn}$  resembles FVSI based on the power flow equation for a transmission line. Continuing from equation 3 above and replacing  $R + jX$  by  $Z < \theta$ , gives an expression for the received reactive power at Bus j:

$$Q_j = \frac{U_i U_j}{Z_{ij}} \sin(\theta - \delta) - \frac{U_j^2}{Z_{ij}} \sin \theta \quad (14)$$

Using the same technique as for FVSI, the receiving-end voltage can be expressed as a second-order equation:

$$U_j^2 \sin \theta - U_j U_i \sin(\theta - \delta) - Q_j Z_{ij} = 0 \quad (15)$$

Similarly as with FVSI, the system remains stable as long as there are only real solutions to equation 9 above.

Rearranging the equation and using the fact that  $Z_{ij} \sin \theta = X_{ij}$  gives the equation for the line stability as:

$$L_{mn} = \frac{4 X_{ij} Q_j}{U_i^2 \sin^2(\theta - \delta)} \leq 1 \quad (16)$$

The similarity of the two indicators can be illustrated by inserting  $\delta = 0$  in equation 10 above to give:

$$L_{mn} = \frac{4 X_{ij} Q_j}{U_i^2 \sin^2(\theta)} = \frac{4 Z_{ij} Q_j}{U_i^2 X_{ij}} = FVSI \text{ for } \delta = 0 \quad (17)$$

Thus the only difference between  $L_{mn}$  and FVSI is that  $L_{mn}$  accounts for the voltage angle difference which FVSI assumes to be zero. This is also the advantage of FVSI over  $L_{mn}$  as it only requires measurements of magnitudes only whereas  $L_{mn}$  requires synchronized phasor measurements at both line ends.

## METHODOLOGY

We shall use the voltage security constrained load flow solution on the IEEE 10 Generators 39 bus test system as shown in Figure below as our base case. The voltage profiles shall be restricted to 0.9 to 1.1 per unit as the security constraint.

PSAT software is used on MATLAB's Simulink platform to obtain the base case load flow solution. This will assist to optimize i.e. to achieve the twin goals of economy and security of the power system voltage. Here, the constraints are the minimization of real power losses, minimization of the cost of active power generation, minimization of reactive power losses for better voltage profiles, maximization of active power transfers and minimizing the cost of installation of the FACTS devices.

The load flow equations shall further be enhanced so as to obtain the voltage stability indices, namely the Fast Voltage Stability Index (FVSI) and the Line Stability Index (LSI). Using the above base case, the two VSI's shall be evaluated for every load bus in the system. The reactive power shall be increased for a chosen load bus gradually until the solution fails to give results/converge for the computable VSI's. After all the load buses are done, the results shall be sorted out for maximum loadability of all the load buses in ascending order with the smallest maximum loadability ranking as the highest which implies the weakest bus in the system. The weakest bus will be the most optimal bus for installation of FACTS devices, namely the UPFC. Finally, the Sparse Matrix Visualization for the system with and without the UPFC shall be obtained.

RESULTS

The load flow solution for the base case converged in 0.388 seconds as shown in Fig. 4. On installation of the UPFC in load bus #4, the load flow solution converged in 0.483s as illustrated in Fig.5. The ranking system shown in Table -1 was developed to determine the stressed and most heavily loaded load buses. The sparse matrix visualization are shown in Fig. 6 & 7.

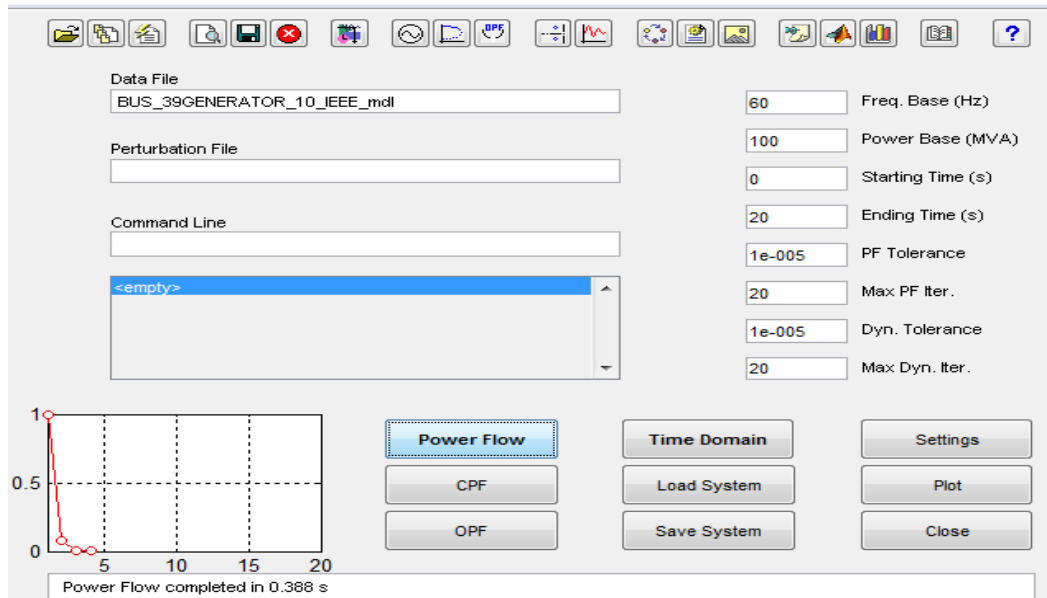


Fig.4 Voltage Constrained Load flow solution (Base Case)

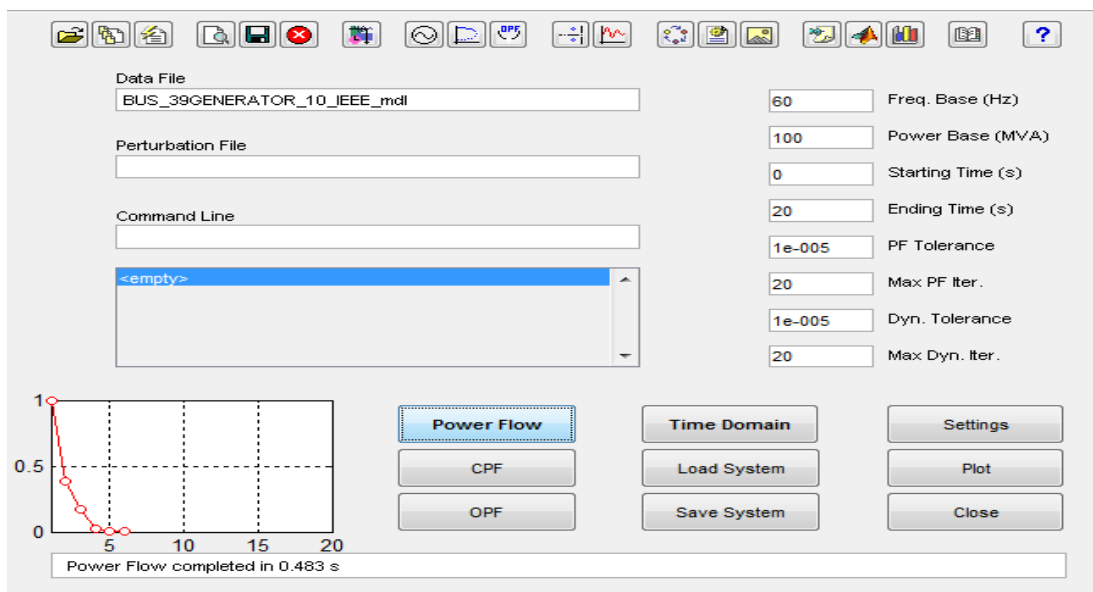


Fig.5 Voltage Constrained Load flow solution on UPFC Installation

Table-1 Load Bus Ranking using the FVSI and  $L_{mn}$  Techniques

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Load bus	4	3	7	8	16	18	15	20	21	12	24	39	25	26	27	31	28	23	29
FVSI	0.9224	0.9125	0.9115	0.9075	0.9029	0.9009	0.8997	0.8901	0.8899	0.8865	0.8725	0.8865	0.8615	0.8556	0.8433	0.8256	0.8166	0.8127	0.8066
$L_{mn}$	0.9429	0.9401	0.9385	0.9276	0.9165	0.9111	0.9008	0.8997	0.8966	0.8911	0.8895	0.8856	0.8735	0.8662	0.8595	0.8487	0.8345	0.8276	0.8215

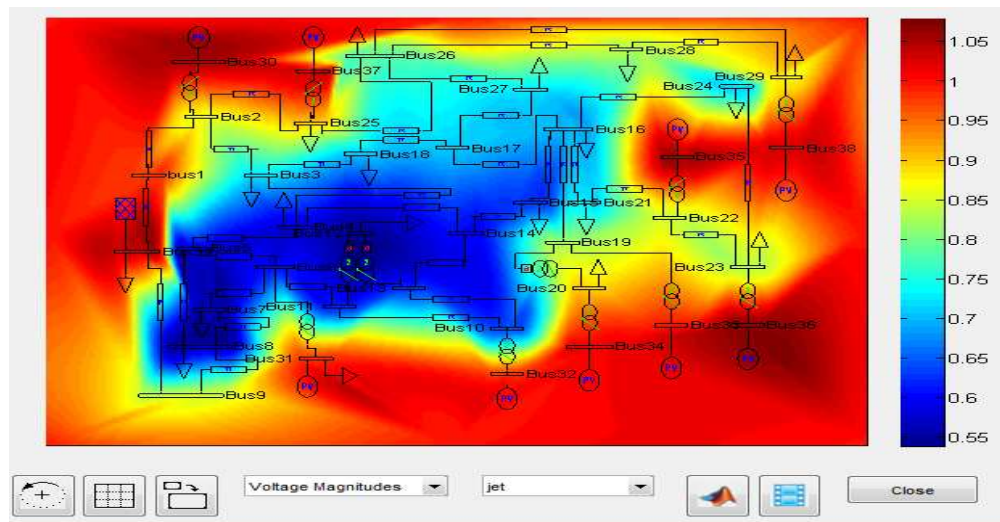


Fig.6 Sparse Matrix Visualization (without UPFC)

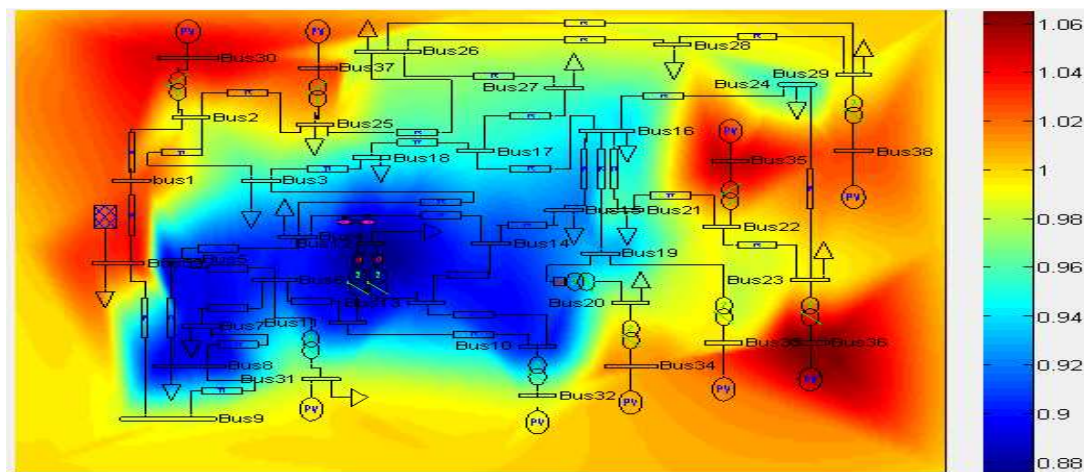


Fig.7 Sparse Matrix Visualization (with UPFC on Load Bus#4)

## DISCUSSION

Using the Voltage security constrained optimal load flow solution alongside (13) and (16) above we computed the FVSI and  $L_{mn}$  respectively for all the nineteen load buses. The results are ranked in Table-1. From the above table, Load Bus#4 is the weakest bus thus the best candidate for optimal placement of FACTS devices for voltage stability improvement. On the other hand, Bus#29 is the most stable buses and the last candidate for voltage support using FACTS devices.

From (16) and (17) above, the results of  $L_{mn}$  are more sensitive than those of FVSI in identification of the weakest buses. This is because  $L_{mn}$  accounts for the voltage angle difference which FVSI assumes to be zero. The results in Table-1 above confirms the said assertion. The Sparse Matrix visualization figures 6 and 7 shows a great improvement on voltage profiles on placement of UPFC on load Bus#4.

The UPFC was devised for the real-time control and dynamic compensation of ac transmission systems, providing multifunctional flexibility required to solve many power system challenges. It is a generalized synchronous voltage source (SVS) represented at the fundamental frequency by voltage phasor  $V_i V_j$  with a controllable magnitude  $0 \leq V_i V_j \leq V_i V_j \max$  and angle  $0 \leq \rho \leq 2\pi$  in series with a given transmission line. The SVS generally exchanges both reactive and real power with the transmission line. The wide range of control for the transmitted power that is independent of the transmission angle  $\delta$  indicates not only superior capability of the UPFC in power flow applications. This is illustrated in (3) and (4) above thus the marked improvement in the voltage profiles as per Fig.7 above.

## CONCLUSION

The objective of using optimally placed FACTS devices for voltage profile improvement was well achieved. Improvement of voltage stability in real time was achieved as per the main objective of the research work.

There is a need for continuous research on the use of FACTS devices for improvement of various power system parameters as networks become more and more complex and strained across the world. A good area of future research work is on the optimal combination of several FACTS devices such as the UPFC and the Inter Line Power Flow Controllers (IPFC) for voltage profile improvement.

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