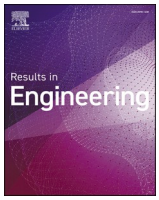
Results in Engineering 21 (2024) 102024



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Numerical modelling of Ti6Al4V(ELI) hierarchical honeycomb structures of order one at the vertices

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A B S T R A C T

Lattice structures are used in engineering industries because of their good mechanical properties, including high stiffness, high strength, as well as specific energy absorption and fracture toughness. They are often built with hierarchical structuring, which adds complexity to them, while improving their mechanical properties. Unfortunately, there is limited literature available regarding how hierarchical structuring affects the stress levels, stress concentration, and deflection of hierarchical-lattice structures. In this work, Ti6Al4V (ELI-Extra low interstitial) hierarchical honeycomb (HC) models of order one at the vertices comprising hexagonal, circular, and triangular polygons, were analysed numerically. The effects of hierarchical configuration on stress levels, stress concentration, and deflection of the hierarchical HC models were compared to those of the regular HC model, for the case of out-of-plane loads in uniaxial compression tests. In terms of load-bearing capacity and stiffness for loads applied in the in-plane x-direction, the hierarchical HC model with hexagonal substructures at the vertices ranked higher than the models with circular or triangular substructures. For loading in the in-plane y-direction, the hierarchical HC model with circular substructures at the vertices ranked highest. The highest load-bearing capacity and stiffness for loading in the in-plane and out-of-plane directions was observed for in-plane loading in the x-direction. Deflections were smaller for loading in the two in-plane directions than in the out-of-plane direction. In-plane loading at the vertices of these structure gave rise to high stress concentrations at the points of loading. Loading the numerical models in the plane, directly at the apex regions led to the highest magnitudes of stress and deflection. Hierarchical honeycomb structures with lower sharp changes of geometry at the vertices minimized the concentration of stresses there.

1. Introduction

In the aerospace, automotive, and biomedical industries, additively manufactured thin-walled lattice structures are extensively implemented for strain energy absorption. This is because of their good mechanical properties, which include high specific stiffness and strength [1], as well as specific energy absorption [2,3]. One of the most extensively researched type of lightweight structures is the polygonal cell-based lattice structure [4–7]. Conventional cell-configurations for creating polygonal cell-based lattice structures include the regular-hexagon, circle, rectangle, and triangle [4]. Such structures have been studied using analytical, computational, and experimental approaches [4–12]. Methods for enhancing the absorption of strain energy have also been devised and investigated [13–15].

Aerospace industries typically use lattice designs to manufacture aircraft parts such as fuselages, interior panels, parts for rocket propulsion engines, and wing structures [1,16]. The use of lattice parts reduces the aircraft's overall weight, cuts fuel consumption and increases cargo space. Turbine blades, compressor vanes, and combustion chambers are built with specific regions of the parts incorporating lattice designs. This is due to the fact that lattice designs reduce thermal conductivity, have high strength-to-weight ratio and as a result, are ideal for both high operating temperatures and high loads of aircraft engines [16]. Lattice structures are additionally integrated in thermal management systems used in aerospace applications. Given their open-cell architecture, which allows for efficient heat transfer and dissipation, they function well as heat exchangers, cooling panels, and thermal protection systems such as heat shields for re-entry vehicles [17–19]. In automotive fields [20–24], lattice structures are used for building lightweight vehicle structures such as chassis, frames, and body panels. Automobile manufacturers apply lattice designs to reduce the weight of vehicles, which results in lower fuel consumption and gas emissions, higher manoeuvrability, and increased load-carrying capacity [20]. In addition, lattice parts are useful in car safety systems because they are capable of absorbing and dissipating impact energy. They are typically used to design crashworthy parts such as impact-absorbing bumpers, safety cages, and reinforced structures with the intent to improve passenger safety during accidents. In vehicle suspension systems, lattice structures provide strength, stability, and weight saving [21,22,24]. They are additionally applied in the manufacturing of lightweight

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https://doi.org/10.1016/j.rineng.2024.102024
Received 27 October 2023; Received in revised form 4 February 2024; Accepted 10 March 2024
Available online 13 March 2024
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suspension components such as control arms and knuckles, which improve vehicle control, stability, and driving comfort [20,24]. In tissue engineering and regenerative medicine fields, lattice designs are used to build scaffolds that mimic the natural extracellular matrix [4]. They provide three-dimensional scaffolding to promote the growth of cells and regeneration of tissue [4,5]. The permeability of lattice structures promotes the integration tissue and reduces the risk of implant-failure [25]. Furthermore, they allow for the transfer of nutrition and oxygen [4]. Lattice structures are also used to build orthopaedic implants including bone plates, screws, and hip replacements that provide mechanical stability and promote the growth of tissue and its integration over time [4,25]. Because lattice parts reduce weight while maintaining mechanical strength and rigidity, they are ideal for surgical instruments and tools. Lattice-structured surgical equipment, including forceps, retractors, and trays, improve the lightweight design and reduce fatigue during procedures [25,26].

When hierarchical structuring is introduced in lattice designs, the mechanical properties are expected to improve [4]. Hierarchical structures typically occur in biological materials across a wide range of length scales, including wood, bone, tendon, beetle elytron tissues, muscle, and spider web [27]. Numerous studies [28–33] have shown that the intricate structural hierarchy is of paramount importance for biological materials to have good mechanical properties, such as high specific stiffness, specific strength, and energy absorption efficacy. Hierarchical honeycombs (HCs), inspired by these naturally occurring materials, have in recent years been synthetically introduced into conventional HCs [27]. Numerous studies [4,28,30,32,33] highlight the fact that lightweight hierarchical structuring at all levels could significantly enhance mechanical characteristics and enable tailored performance, resulting in numerous interesting applications for hierarchical structures [4,27]. For instance, the hierarchical honeycomb (HC) is constructed with a large-scale structure and unit cells comprised of small-scale configurations. Provided the large-scale and small-scale configurations are consistent, these HCs are referred to as self-similar hierarchical HCs and conversely as non-self-similar [27]. As shown in Fig. 1 [27], typical hierarchical HCs include cell-wall-based, fractal-based, spiderweb-based and vertex-based hierarchies, depending on the choice of the small-scale configuration. The cell-wall-based hierarchy substitutes the primary cell wall of a large structure with a small-scale counterpart. Tan et al. [34] presented hierarchical re-entrant HCs, built by substituting the cell wall of auxetic HC with conventional hexagonal and triangular topologies. The numerical simulation results obtained by the authors revealed that as compared to regular re-entrant HC, the specific energy absorption of hierarchical HC was increased by approximately 292%, as was the mean crushing load, in a quasi-static condition. Zhang et al. [35] suggested a cell-wall-based hierarchical triangular HC that was built by generating a series of identical sub-triangular configurations on the original cell wall of a standard triangular HC. The hierarchical HC outperformed standard HC with regard to crushing load and energy absorption, suggesting more stable deformation. Fractal-based hierarchy implies that similar small-scale configurations are arranged within the large-scale arrangements. Tsang et al. [36] presented a fractal-based hierarchical circular HC modelled by mimicking the muscle tissue. The obtained results showed that the impact energy absorption capacity had been significantly enhanced with further improvements observed at higher-level hierarchies. Zhu et al. [37] generated a hierarchical hexa-chiral structure to minimise the distortion of the central circle in hexa-chiral configurations, in which the regular hexa-chiral configuration reinforced the central circle of the large-scale hexa-chiral structure. When compared to the original hexa-chiral HC, this structure's mechanical properties were significantly higher. The spiderweb-based hierarchy, as the name suggests, is inspired by the spider web, which is a radial configuration of small-scale structures from center to the outside. He et al. [38] developed a spiderweb-based hierarchical HC wherein the smaller hexagonal structure was integrated into the central basic unit cells for building a hexagonal network. The results obtained from numerical analysis of this structure revealed that the specific energy absorption of first and second-order hierarchical HC increased by 62.1% and 82.4%, respectively.

Past studies have shown that a hierarchical design approach that substitutes the original material at the vertices of a large-scale configuration with a small-scale configuration improves the structure's ability to absorb strain energy [39–42]. Ajdari et al. [39], Oftadeh et al. [40], Mousanezhad et al. [41], and Zhang et al. [42] produced fractal-like hierarchical honeycomb (HC) structures by substituting each vertex of a normal HC with a small-scale, self-similar hexagon. This is a three-edge vertex substitution process that could be performed at lower scales to construct fractal-looking structures having higher structural hierarchical orders. According to Ajdari et al. [39], the 1st and 2nd hierarchical orders improved in-plane stiffness by 2.0 and 3.5 folds, respectively. Oftadeh et al. [40] investigated the mechanics of higher-order hierarchical HC structures and found that the 1st to 4th orders of hierarchical HC structures are 2.0–8.0 times stronger than their conventional hexagonal counterpart. Mousanezhad et al. [41] studied the effect of hierarchy and imposed deformations on HC structures. The results obtained highlighted the fact that hierarchical structuring of topologies and instability-induced alterations of the arrangement of HC structures under compression loading could be used to effectively tailor the gaps between bands and the directional changes of phononic crystals. This investigation provided insight into the function of structural arrangement and hierarchy in determining the dynamic behaviour of phononic crystals, as well as the prospects for building tailored phononic devices with high absorptions of strain energy. Zhang et al. [42] observed that lowering the wall thickness while replacing the vertices of a regular HC structure resulted in increased absorption of strain energy for the same relative density. Fig. 2 shows the evolution of a regular HC structure when a hexagonal hierarchical order is introduced at the vertices and increased.

Except for limited work on vertex-based hierarchical HC structures produced by substituting vertices with non-self-similar substructures [41,43,44], most research on hierarchical structures has concentrated on self-similar substructures. This is due to a gap in knowledge in defining the exact mathematical models used to determine the geometric parameters of hierarchical HC structures with non-self-similar substructures. Most research on the behaviour of hierarchical lattice structures has concentrated on their fracture toughness, absorption of energy, and specific strength [4,39,40,42]. Moreover, research on the influence of different configurations of vertex-based hierarchical lattice structures on stress levels, stress concentration, and deflection is lacking. To address this research gap, the authors aim to provide new insights in this paper, by investigating the behaviour of non-self-similar Ti6Al4V (ELI-extra low interstitial) hierarchical lattice structures under load. In contrast to past research, which focused on self-similar substructures, different domains of non-self-similar substructures at the vertex level are considered in this paper. Furthermore, analysis of these structures is carried out to increase understanding of the effects of stress levels, stress concentration patterns and deflection characteristics on the mechanical response of the hierarchical lattice structures studied here. This study also provided a new perspective and ranking of the hierarchical lattice configurations in terms of the structural performance aspects stiffness and load bearing capacity, and this way paves the way for more thorough and complicated analyses in the engineering field.

This paper reports on the development of planar vertex-based hierarchical HC structures comprising hexagonal, circular, and triangular polygons at the vertices. The effects of their geometrical configurations on stress levels, stress concentration, and deflection of each hierarchical HC structure are investigated. The planar hierarchical HC configurations are initially constructed in the materials and methods section based on their current relevant analytical modelling limitations. Following that, finite element (FE) models are created. The numerical results are provided, compared, and analysed in sequence, based on the stress levels, stress concentration, and deflection of the planar hierarchical HC

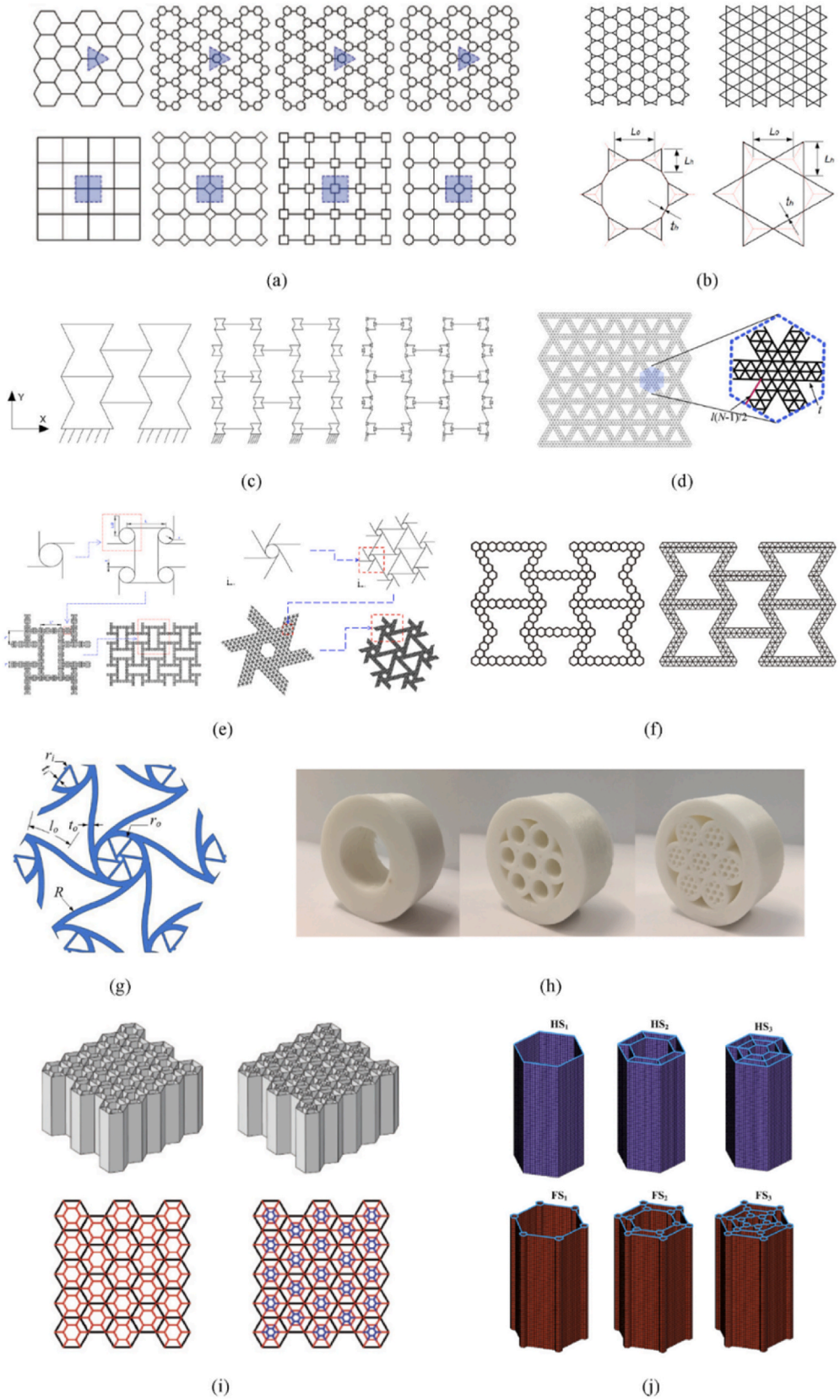


Fig. 1. (a–b) vertex-based hierarchical polygonal structures, (c) vertex-based hierarchical re-entrant HCs, (d) cell-wall-based hierarchical triangle HC, (e) cell-wall-based hierarchical using tetra-chiral and hexa-chiral HCs, (f) cell-wall-based hierarchical re-entrant HCs, (g) fractal-based hierarchical hexa-chiral HC, (h) fractal-based hierarchical circular polygon structures, and (i–j) spiderweb-based hierarchical hexa-chiral HCs [27].

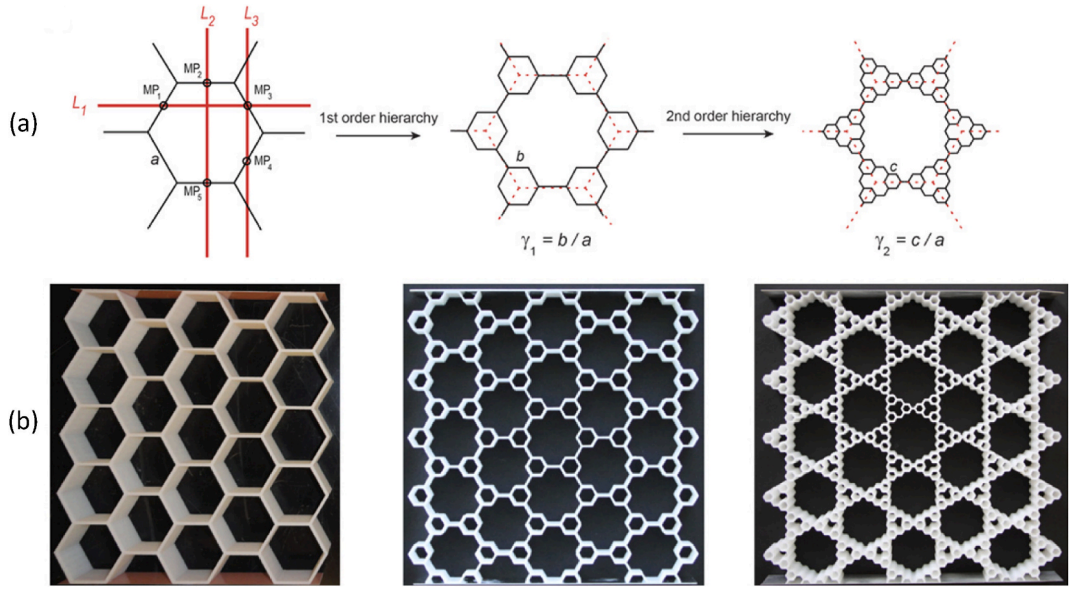


Fig. 2. The geometry of a hierarchical HC (a) a hierarchical HC unit cell with a regular structure and first and second order hexagonal hierarchies shown from left to the right (b) images of HC hierarchical structures produced through additive manufacturing process [39].

structures of order one at the vertices, employing hexagonal, triangular, and circular polygons. The most significant results are then highlighted.

2. Materials and methods

2.1. Hierarchical design configurations

On the basis of the planar regular HC structure, first order vertex-based hierarchical structures were constructed. The HC structure has basic geometric parameters, such as the sloping angle $\theta = 30°$, cell wall thickness t, and cell wall length l, shown in Fig. 3 [44].

The parameter t is designed to decrease in value when the same design parameters are assigned to the first order planar vertex-based hierarchical HC configuration. The parent structure in Fig. 3(b) and its hierarchical configurations are related based on the geometrical features shown in Fig. 4(a) and (b), and 4(c) [44]. The circular polygon substituted at the vertex was defined as the smallest circumscribed circle of the hexagonal and triangular substructures using a radius parameter r. This in turn yields the relationship seen in Fig. 4(d) [44]. The lengths of the walls within the hexagonal substructure, therefore, are related as follows:

$$l_{12} = l_{23} = l_{34} = l_{45} = l_{56} = l_{61} = r \tag{1}$$

Similarly, the lengths of the cell walls within the triangular sub-structure are related as follows:

$$l_{13} = l_{35} = l_{51} = r\sqrt{3} \tag{2}$$

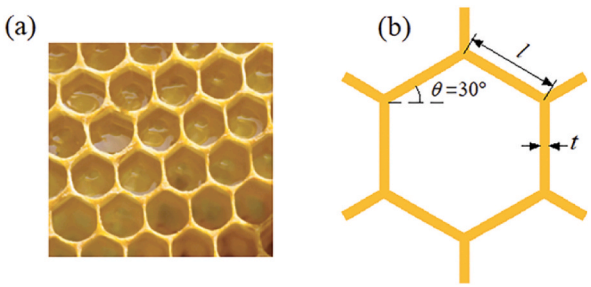


Fig. 3. (a) Natural beehive HC and (b) regular hexagonal geometry of a HC [44].

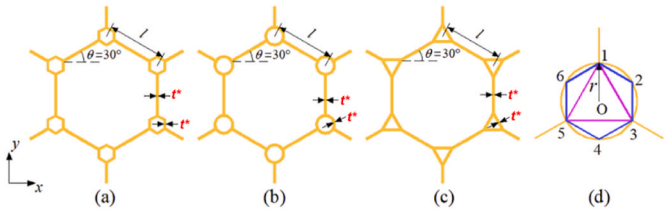


Fig. 4. (a) Hexagonal substructures, (b) circular substructures, (c) triangular substructures and (d) a diagram illustrating the relationship between these three different substructures [44].

The relative density, $\overline{\rho}$, is the ratio of the structure's density to that of the reference material for the cellular structure, and it is expressed approximately as:

$$\overline{\rho} = \frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t}{l} \tag{3}$$

The symbols (ρ^*) and (ρ_s) stand for, density of the regular HC structure and density of the reference solid material, respectively.

Planar regular and hierarchical HC structures were produced with the same relative density of $\overline{\rho} = 0.10$ and base hexagonal edge length of $l_o = 20$ mm. This setting of the relative density was because structures with a relative density greater than 30% of the parent solid structure are no longer categorized as cellular structures, but rather as solids with separated holes [45]. The thickness of the cell walls of the planar hierarchical HC structures was lowered to preserve overall relative density. This was accomplished by using a structural configuration parameter, obtained from pre-existing analytical models that have been published [39,42,46]. From this, the corresponding configuration for the 1st order planar vertex-based hierarchical HC configuration was predicted as follows:

$$\gamma_{(h,c,t)} = \frac{l_{(h,c,t)}}{l_o} \tag{4}$$

The symbols l, and l_o are the lengths of the substructure and base hexagon edges, respectively, and parameter γ denotes the function of the ratio of these two lengths. The hexagonal, circular, and triangular sub-structures are represented by the subscripts h, c, and t, respectively.

Geometric constraints of $0 \leq \gamma_h \leq 0.5$, $0 \leq \gamma_c \leq 0.5$, and $0 \leq \gamma_t \leq \sqrt{3}/2$ were imposed on the first order hierarchical HC structures comprising of hexagonal, circular, and triangular substructures at the vertices, respectively. These constraints have been documented in numerous studies [4,13–21] as being advantageous for designing hierarchical HC structures with acceptable mechanical properties. In addition, hierarchical HC structures built with additive manufacturing (AM) technology under the imposed constraints are predicted to be most accurate in their designs [14,15,39–44]. As a result, the relative density for each of these hierarchical HC structures was determined using the formulae [44]:

$$\frac{\rho_h^*}{\rho_s} = \frac{2}{\sqrt{3}}(1+2\gamma_h)\frac{t_h^*}{l_o} \tag{5}$$

$$\frac{\rho_c^*}{\rho_s} = \frac{2}{3\sqrt{3}}\left[-3\left(\frac{t_c^*}{l_o}\right)+2\gamma_c(2\pi-3\gamma_c)+3\right]\left(\frac{t_c^*}{l_o}\right) \tag{6}$$

$$\frac{\rho_t^*}{\rho_s} = \frac{1}{3}\left(2\sqrt{3}+4\sqrt{3}\gamma_t-4\gamma_t\right)\frac{t_t^*}{l_o} \tag{7}$$

For the purpose of reducing the complexity of the models, the thicknesses of the cell walls of planar hierarchical HC structures, was assumed to be uniform, and therefore were assigned the same value ($t_h^* = t_c^* = t_t^*$) in this study. As a result, the constructed and numerically simulated planar regular HC model had a value of wall thickness $t = 1.75$ mm. The 1st order planar hierarchical HC models comprised of hexagonal, circular, and triangular structures at the vertices had values of the parameter $\gamma_h = 0.3$ and $t_h^* = 1$ mm, $\gamma_c = 0.3$ and $t_c^* = 1\ mm$, and $\gamma_t = 0.58$ and $t_t^* = 1\ mm$, respectively. When predicting the behaviour of 1st-order hierarchical HC structures with good mechanical properties, the aforementioned values of the parameters are often used [14,15,40,42–44]. Fig. 5 shows geometries created in SolidWorks software by employing the foregoing values of parameters for 1st order hierarchical configurations of the regular HC geometry. In the ensuing work, the abbreviations HCR, HCH, HCT, and HCC are used to refer to the regular HC structure and hierarchical HC structures with hierarchical hexagonal, triangular, and circular substructures, respectively.

2.2. Numerical models

2.2.1. Finite element modelling

The effects of different planar hierarchical topologies of the HCR structure on the stress levels, stress concentration, and deflection were studied using finite element analysis. The load-bearing capacity and stiffness of the models were determined under uniaxial compression loads applied to the structures through numerical simulations based on the implicit analysis in ABAQUS. The software package was chosen for its capacity to simulate complex nonlinear material behaviour, large deformations, and contact interactions. These features render ABAQUS preferable for capturing the complexities of hierarchical HC structures, which might exhibit nonlinear behaviour and have intricate geometries. ABAQUS offers powerful meshing capabilities that allow high-fidelity discretization of complicated structures. The capacity to generate a detailed and refined mesh is critical for accurately capturing stress concentrations, deformation patterns, and failure modes in hierarchical

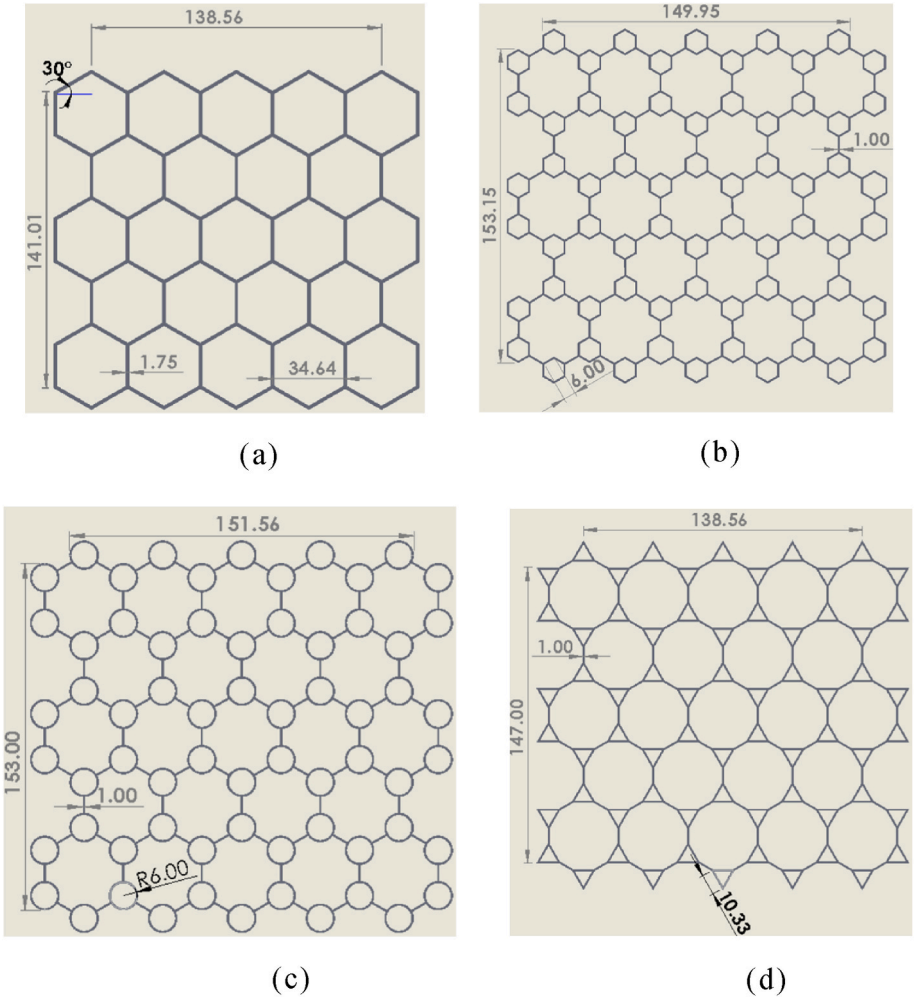


Fig. 5. The built planar geometry of the (a) HCR, (b, c and d) HCH, HCC and HCT, respectively.

HC structures. In addition, ABAQUS software offers parallel processing, allowing for efficient computations on large and computationally intensive simulations. This is particularly advantageous when working with complicated hierarchical structures given that it can significantly reduce simulation time. Fig. 6 shows the setup of the planar numerical models loaded in-plane. This was considered since both HCR and hierarchical HC structures are normally exposed to planar in-plane loading. The planar in-plane loading condition was chosen to provide a basis for comparison of the results obtained from the current numerical models, with the ones existing literature on lattice structures that are primarily used for high-strain energy absorption. Such in-plane loads lead to out-of-plane deflections in the form of buckling, and also lead to deformation of the cell walls and stress concentrations at the vertices [39–46].

The FE models were built using two rigid plates and a planar HCR model sandwiched between them as shown in Fig. 6. One of the rigid plates for each of the models was allowed to only move in the direction of applied load during the compression tests, while the other one remained entirely fixed. A load of 3000 N was applied separately in the x- and y-directions as shown in Fig. 6. This load was chosen to ensure that the structures could be analysed within their elastic phase. This in turn, allowed the expected outcomes to be predictable within the design constraints of typical engineering applications. The interfaces between the two rigid walls and each of the models were assigned a frictionless contact and the normal interaction property was assigned as a hard contact to prevent the model from penetrating the rigid surface. The planar HCR structure was meshed using hourglass-controlled, 8-node, reduced integration quadratic shell elements, S8R, with five integration points across the wall thickness. Recent work by the authors [46] showed that when the element size is lowered to 0.9 mm, the simulated average compression stresses attain convergence for the HC model. To minimise the computation resources, the global element size of 0.9 mm was adopted for all models. The authors performed mesh convergence analysis in prior research [46], and observed that the numerical findings were not influenced significantly by mesh sizes below 0.9 mm. The cell wall material was assumed to be elastic during numerical simulations.

The HCR structure was assigned the properties of a quasi-statically loaded Ti6Al4V(ELI) alloy with an elastic modulus of $E = 113.8$ GPa, yield stress of $\sigma_{ys} = 790$ MPa, Poisson's ratio of $\nu = 0.342$, and density of $\rho_o = 4430$ kg/m^3 [47,48]. This numerical modelling setup was used to predict the resultant stress levels, stress concentrations, and deflections of the HCR structure. The same procedure was adopted for developing the rest of the numerical models, including the planar hierarchical HC structures discussed in the ensuing analysis. The FE model for the HCR, and HCH, HCC, and HCT structures were simulated based on Hooke's law expressed below.

$$\sigma_V = E\varepsilon \tag{8}$$

The symbols σ_v, E and ε represent the longitudinal stresses, elastic modulus of the material and the strain of the model, respectively. In this study, the resulting maximum stresses were limited to linear elastic analysis.

3. Results and discussion

3.1. Numerical results of stress levels and stress concentration

Figs. 7 and 8 show von Mises stress contours generated by applying a 3000 N load to the HCR model, and HCH, HCC, and HCT models. These contour plots show the stress levels and contour maps of stress distribution obtained for loading in the x- and y-directions separately. Figs. 7 and 8 show that the hierarchical HC models subjected to loading in the x-directional had the lowest values of maximum von Mises stress. This is because the loading along the x-direction was applied along the lengths of three members (larger contact areas), while along the y-direction it was applied at five apex points (smaller contact areas).

It is garnered from the stress contour keys in Fig. 7 that when a load is applied in the x-direction, the HCH model has the lowest value of maximum von Mises stress (approximately 12.37% lesser than the value obtained for the HCR model). The HCC model records the second lowest value of maximum von Mises stress (about 3.99% decrease from that of the HCR model), whereas the HCT model is ranked last. The HCT structure recorded the highest value of maximum von Mises stress because, compared to the other two geometries, it was loaded directly at the apex regions. These are regions with sharply changing geometries. The apex regions are highly stressed regions that are liable to fail even at low applied loads. Instead of the maximum von Mises stress level decreasing for the HCT model, as observed for the other two models, they increased by 48.43% with reference to the HCR model.

Fig. 8 shows that with the load applied in the y-direction, the HCC model attained the lowest value of maximum von Mises stress (approximately 11.8% lower than the value obtained for the HCR model). For the same loading direction, the HCH model now ranked

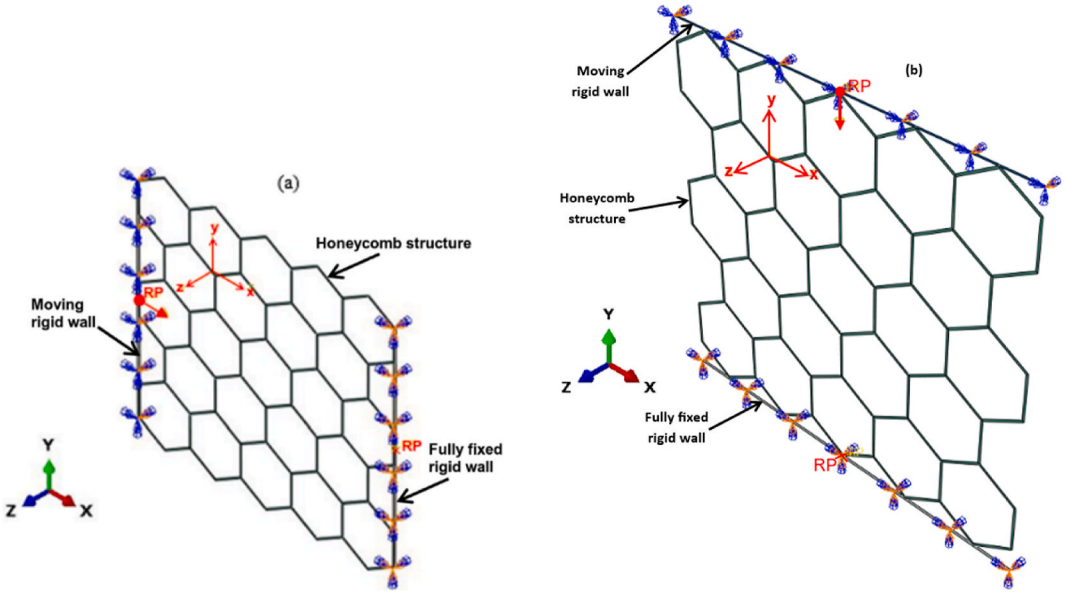


Fig. 6. The planar FE model with load applied along (a) the x-direction and (b) the y-direction.

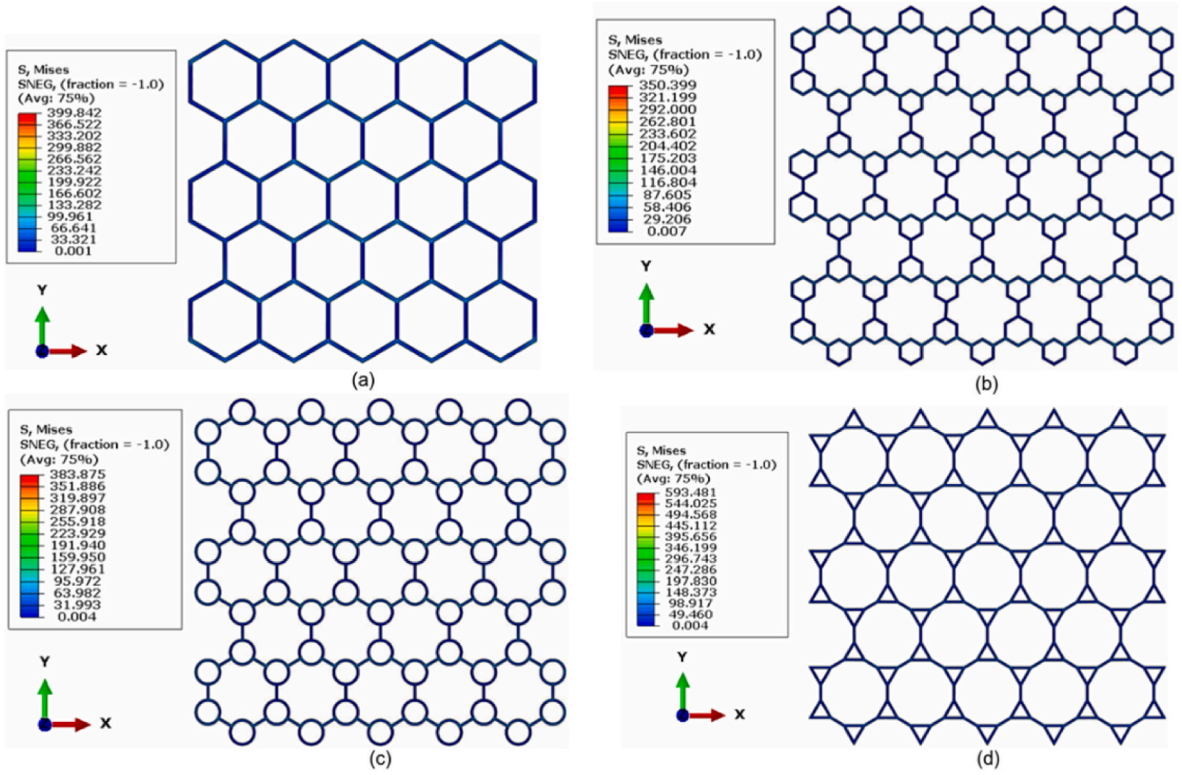


Fig. 7. The von Mises stress distribution for (a) HCR structure and (b), (c) and (d) HCH, HCC, and HCT structures, respectively, for the case of a load applied in the *x*-direction.

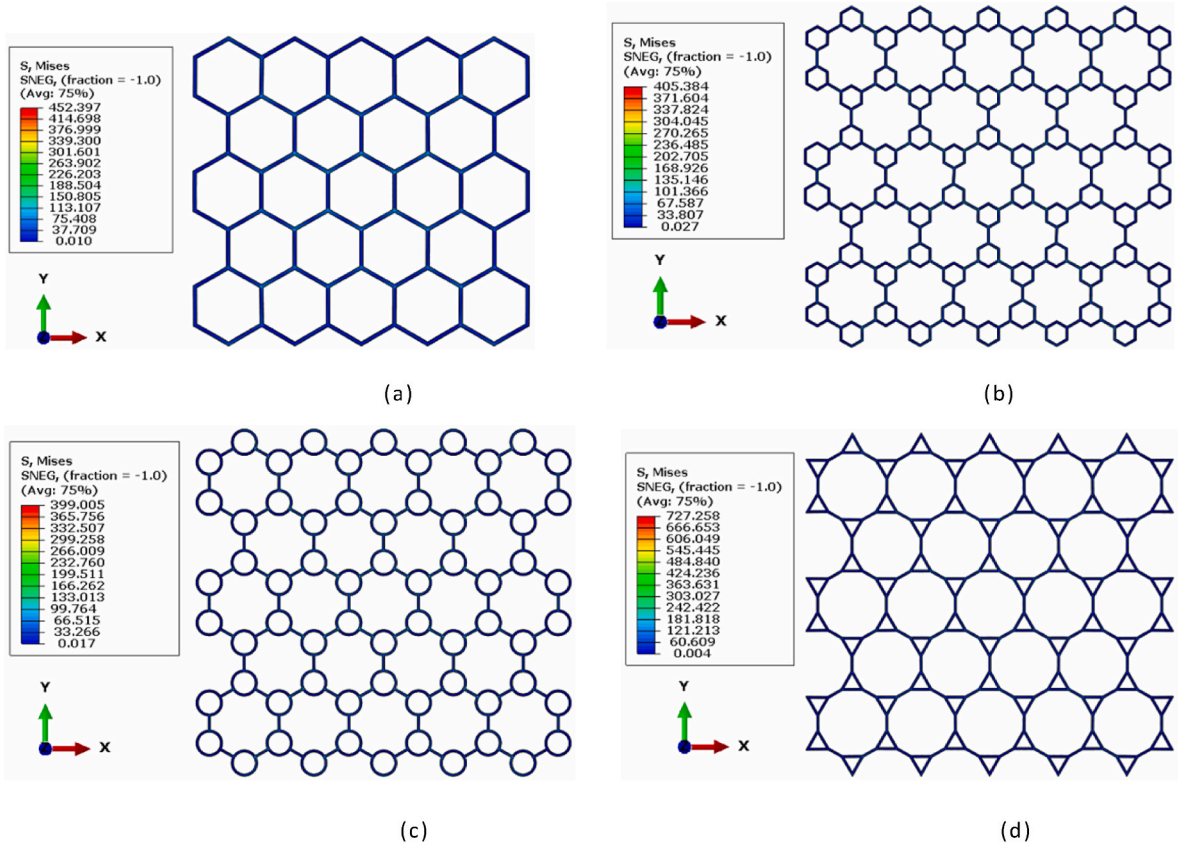


Fig. 8. The von Mises stress distribution for (a) HCR structure and (b), (c) and (d) HCH, HCC, and HCT structures, respectively, for the case of a load applied in the *y*-direction.

second in terms of the generated value of maximum von Mises stress (a decrease of about 10.39% with reference to the HCR model). With reference to the value of maximum von Mises stress obtained for the HCR model subjected to y-directional loading, the HCT model showed an increase of 60.76%. The application of von Mises stress to lattice structures provides for easier and straightforward assessment of yielding, particularly when dealing with complex stresses arising from the use of intricate geometries. In the present investigations, the analysis was conducted with the aim of ensuring all induced stresses were solely elastic. Thus, analysis using von Mises stress was useful in easily identifying the loads beyond which the material was expected to yield.

According to the work discussed in the foregoing three paragraphs, the HCH model had the lowest value of maximum von Mises stress when loaded in the x-direction and the second lowest value of maximum von Mises stress when loaded in the y-direction. The percentage changes for the maximum von Mises stress of the HCH model loaded in all directions separately were greater compared to the differences for the HCC model. In comparison to the HCR model, the two models alternated in ranking. For loading in all directions separately, the HCT model ranked last. Clearly, the HCH model was the best alternative of the three hierarchical structures modelled here.

Apart from the contour keys in the preceding two figures, it is not possible to distinguish any distribution of stress in their walls from them. Therefore, to study the stress distribution along the very thin-walled cell-configurations of the HC structures, sections were extracted and enlarged. It is evident from Figs. 9–12 that enlarging the unit cells gives clear details of von Mises stresses in the numerical models. These figures show that the presence of von Mises stress concentrations at the vertices.

It is evident from Figs. 9–12 that, the structure with the sharpest geometries at the vertices in contact with the rigid plates used to apply loads on the structures had the greatest value of maximum von Mises stress. Since the loading plates were rigid, the induced stresses were much more likely to be concentrated directly at the corners of structures with sharp angles at which contact with the plates was made. This is observed for the HCT model loaded in the x- and y-directions separately, as well as the HCR model loaded in the y-direction. However, as shown in Figs. 9–12, the other hierarchical structures experienced maximum values of von Mises stresses at other sharp corners that were not in direct contact with the rigid surfaces. This is likely due to the restraining effect of the cell edges, which becomes greater as one moves closer to the centers of each structure. Work done in Refs. [49–51] supports this observation. The HCT model has the sharpest corners (with acute angles of 60°) in contact with the rigid plates on both ends, for loading in either one of the two directions. This implies that it is the most stressed and, as a result, is unlikely to withstand the same magnitude of loads as the other hierarchical structures, when loaded along either one of the two

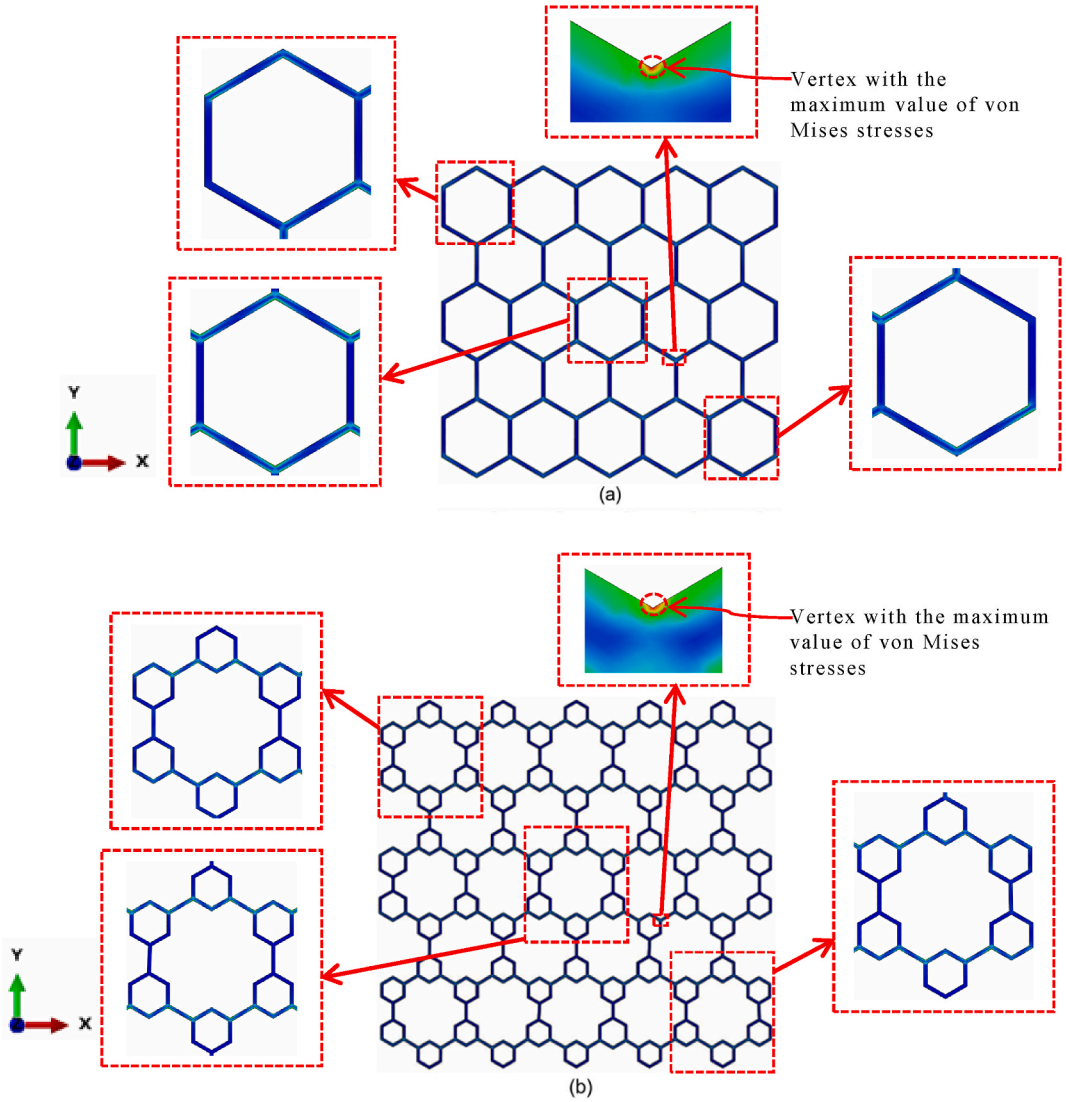


Fig. 9. Stress distribution in (a) HCR model, and (b) HCH model, when loaded in the *x*-direction.

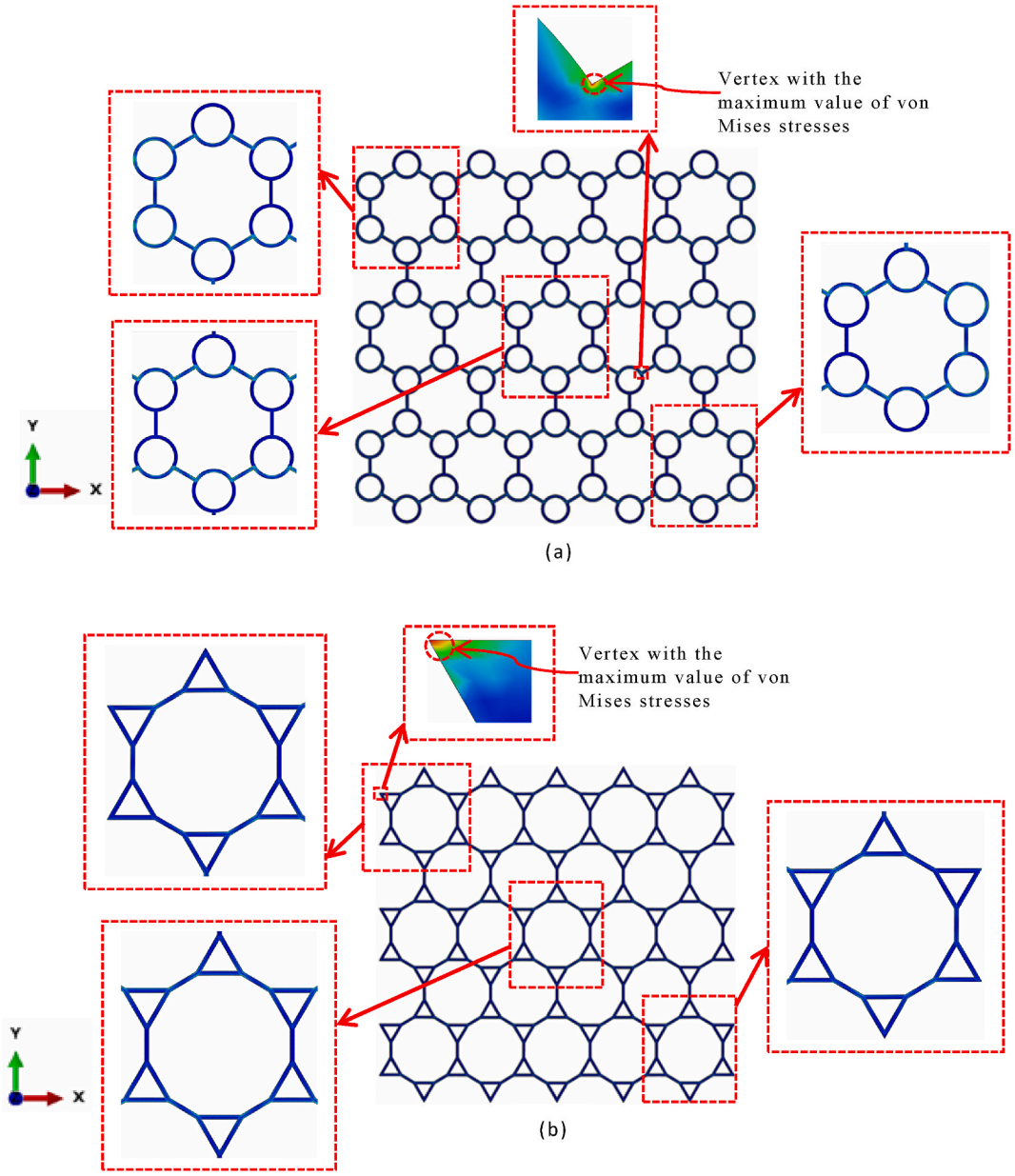


Fig. 10. Stress distribution in (a) HCC model, and (b) HCT model, when loaded in the *x*-direction.

planar directions. Therefore, for improved mechanical performance, reducing sharp changes in geometry at vertices in contact with rigid bodies should be prioritised in the design of such engineering structures.

The magnitude of stresses at the vertices of loaded HCR, HCH, HCC, and HCT models are seen from the foregoing discussion to be a function of their geometric configuration. The vertices have been shown to be highly stressed and are prone to failure under lower applied loads in consistency with the observations made in Ref. [49]. As is known for stress concentration, the sharper the changes of geometry at the structure's vertices, the higher the concentration of stresses in such areas [52], and thus the higher the limitation of such structures to withstand high loads.

From the foregoing discussion, it is deduced that the HCH model attained the lowest maximum von Mises stress when load was applied in the x-direction. This is because the loading plates were resting along the structure's members, distributing the applied load over a larger area as compared to other forms of hierarchical HCs modelled. Thus, this type of hierarchical HC structure is expected to be able to withstand the highest loads of all other structures considered here. The HCC model had the lowest maximum von Mises stress for loads applied in the y-direction. This is because it was the only model amongst those studied here, that had no sharp change of geometry at regions in contact with loading plates, with respect to the y-direction load. Therefore, for loading in the y-direction, this structure is expected to withstand the highest loads amongst the structures studied here.

3.2. *Numerical results of deflection*

Figs. 13 and 14 shows the deflections of numerically modelled HCR, and HCH, HCC, and HCT structures. When these structures were subjected to load in the x-direction, as shown in Fig. 13, the HCH model exhibited the lowest value of deflection. Under y-direction loading, as shown in Fig. 14, the HCC model achieved the lowest value of deflection.

It is seen in Fig. 13 that for a load applied in the x-direction, the maximum value of deflection in each of the hierarchical HC models is lower than the maximum value of deflection for the HCR model. The HCH, HCT, and HCC models attained 21.59%, 6.11%, and 4.07% reduction of their maximum deflections, respectively, with reference to

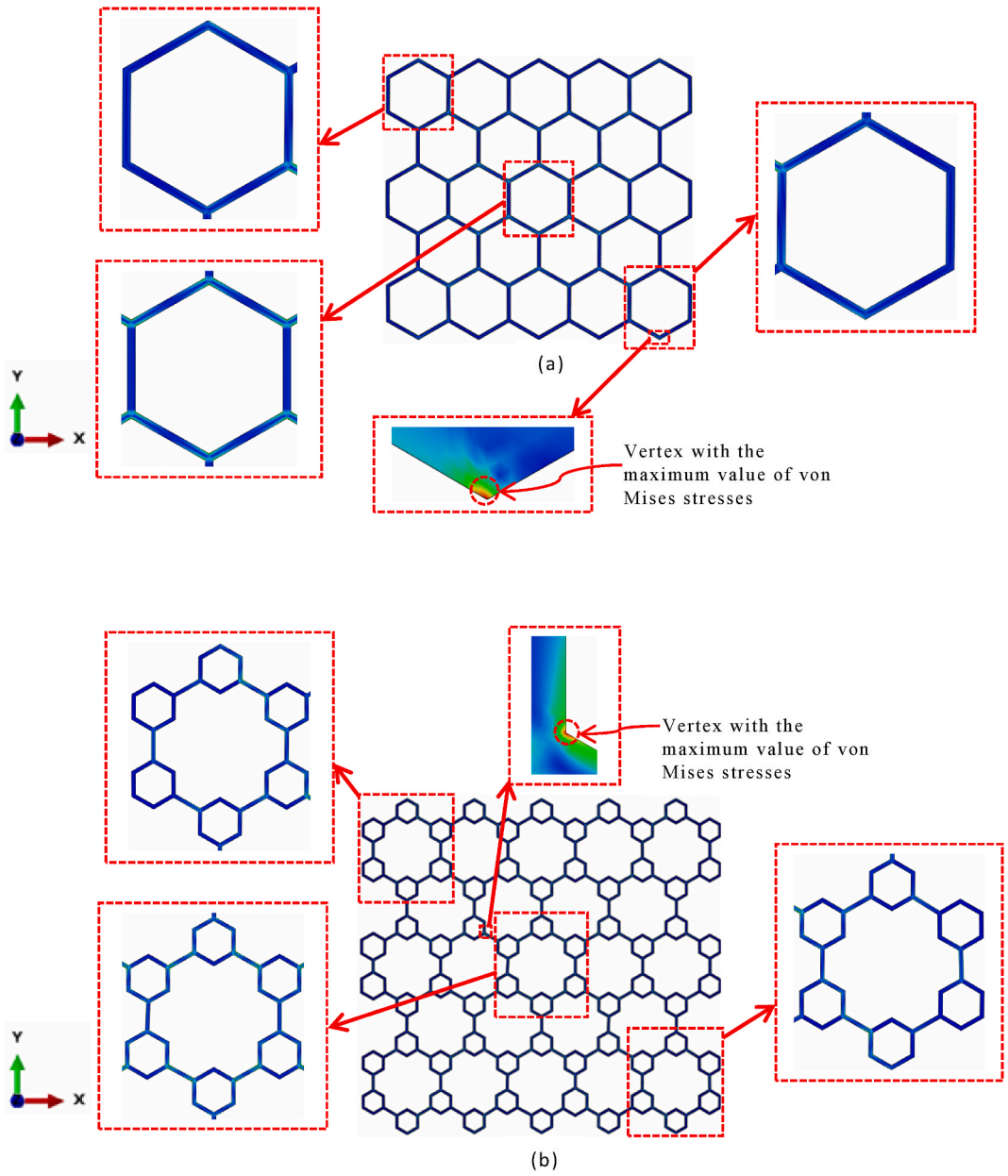


Fig. 11. Stress distribution in (a) HCR model, and (b) HCH model, when loaded in the *y*-direction.

the maximum deflection of the HCR model. For the case where the load was applied in the y-direction, as shown in Fig. 14, the HCC model recorded a lower magnitude of maximum deflection than the HCR model by about 5.05%. The HCH and HCC models, on the other hand, recorded an increase in the magnitude of maximum deflection over that of the HCR model by 0.68% and 14.25%, respectively.

The stiffness of a structure is significant in engineering applications as stiffness is inversely related to deflection [26]. The stiffest structure under-load applied in the x-direction, according to the results obtained in this study, is the HCH model. The HCC model has been shown here to be stiffer in compression for the load applied along the y-direction.

3.3. Comparison of experimental studies in literature and modelling implemented here

Comparative analysis of the numerical models implemented here, and experimental results reported in literature with regard to the behaviour of hierarchical hollow structures could provide valuable insights into the scientific accuracy and consistency in adopting predictive methods to investigate the behaviour of complex structures.

The numerical models in the current work show that loading hierarchical HC structures in the direction of load transfer with larger areas of contact yields lower values of maximum von Mises stresses than loading in the direction of load transfer at smaller areas of contact, such as the apexes where the vertices converge. Lower values of maximum von Mises stresses suggest that the structure has a higher load-bearing capacity compared to structures with higher values of maximum von Mises stress [51–53]. The authors were, however, not able to validate with experimental results, the phenomenon mentioned in the sentence before the previous one, as the available literature on hierarchical HC structures of various polygon shapes at the vertices [13–15,39–44,54] concentrates only on loads applied transversely to the structural members of these structures. It is observed that all these studies have omitted the outermost sharp vertices formed in the basic designs of the structures in an attempt to generate members that are configured transversely to the loads, as opposed to the regular sharp inclined or arc shaped

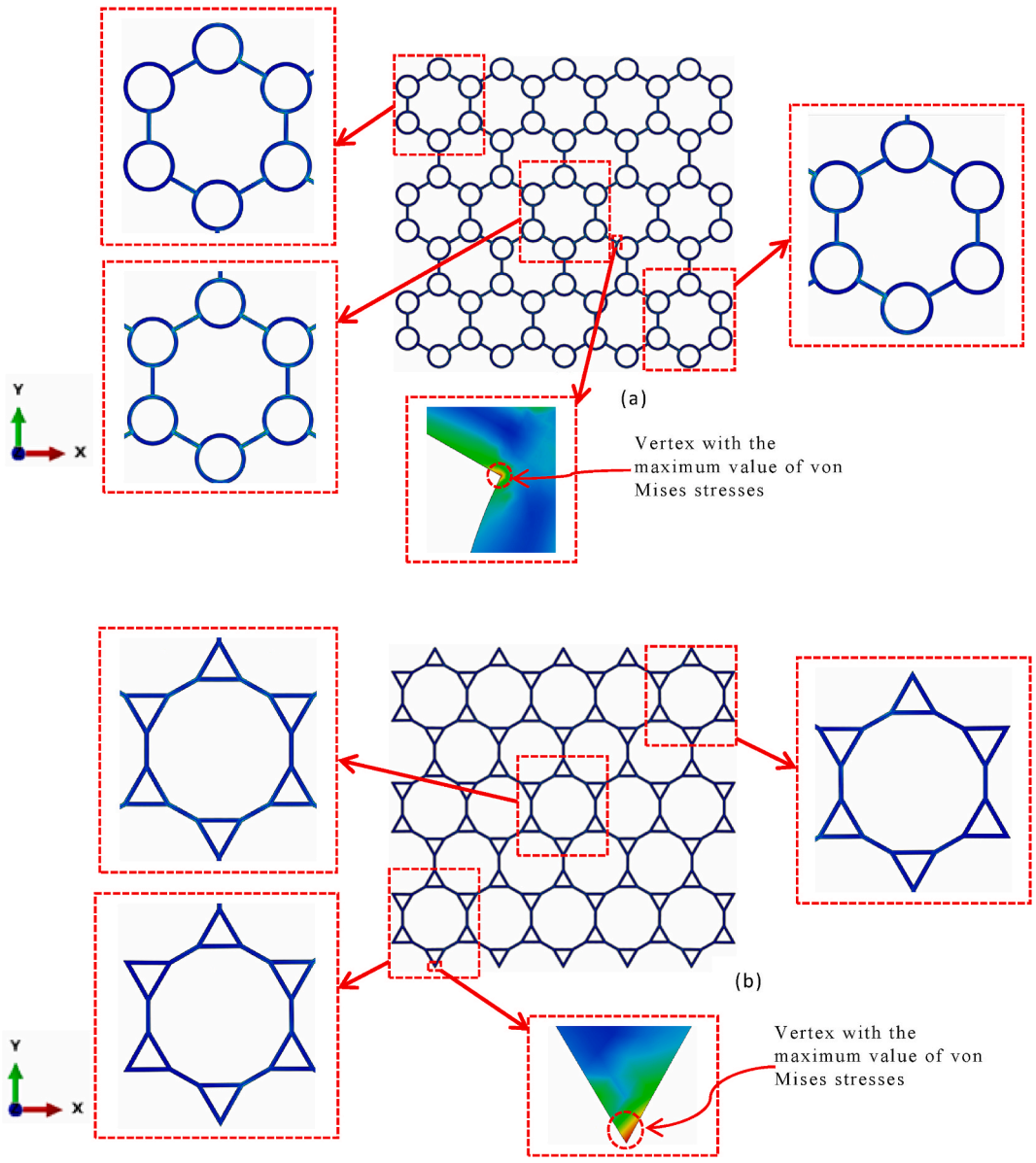


Fig. 12. Stress distribution in (a) HCC model, and (b) HCT model, when loaded in the *y*-direction.

members that were adopted in the present work for the HCT and HCC models, respectively. The observation that loading hierarchical hollow structures in the direction of load transfer with larger contact areas results in lower maximum von Mises stresses is consistent with fundamental structural mechanics concepts discussed in Refs. [51,55,56]. This analysis presented in these references highlighted the need to evaluate direction load transfer and optimise contact regions to improve structural performance. The relationship between lower levels of stress and higher load-bearing capacity is crucial for designing lattice structures with optimal strength. Validation of the numerical results with experimental findings is a shortcoming in existing literature. As mentioned earlier in the paragraph, numerous studies [39–44] focusing on loads applied transversely to lattice parts while ignoring the outermost sharp vertices leads to shortcomings in the experimental methodologies used for validation. This underscores the importance of conducting more extensive experimental studies that consider numerous types of loading conditions and geometric configurations to confirm numerical results. The difference between the approach used in the current study (using regular sharp inclined or arc-shaped members) and prior research [39–44,51,55,56] (configuring members transversely oriented to the applied loads) highlights the need to take geometric complexities into account when conducting experimental tests. Numerous studies [57–64], have highlighted fact that the choice and configuration of the structural parts has significant effects on mechanical properties, as was found in the present work. This finding necessitates an overhaul of experimental approaches in lattice structure research to ensure that they capture the intricacies of general engineering applications accurately. Numerical models provide useful insights; however, the absence of accurate experimental confirmatory tests raises concerns with regard to the applicability of the numerical results in general engineering conditions. Engineers and researchers in the field of lattice structures ought to be cautious when using numerical findings alone in general engineering applications without experimental validation. This is particularly important when working with complicated hollow parts and loading conditions. This identified gap in experimental literature, together with the recognition of limitations in prior studies [39–44,57–64], implies a need for a shift in how experimental investigations are carried out in the field of hierarchical hollow structures. The requirement to replicate outermost sharp vertices and configurations that correspond to general engineering loading conditions must be met. This is expected to have

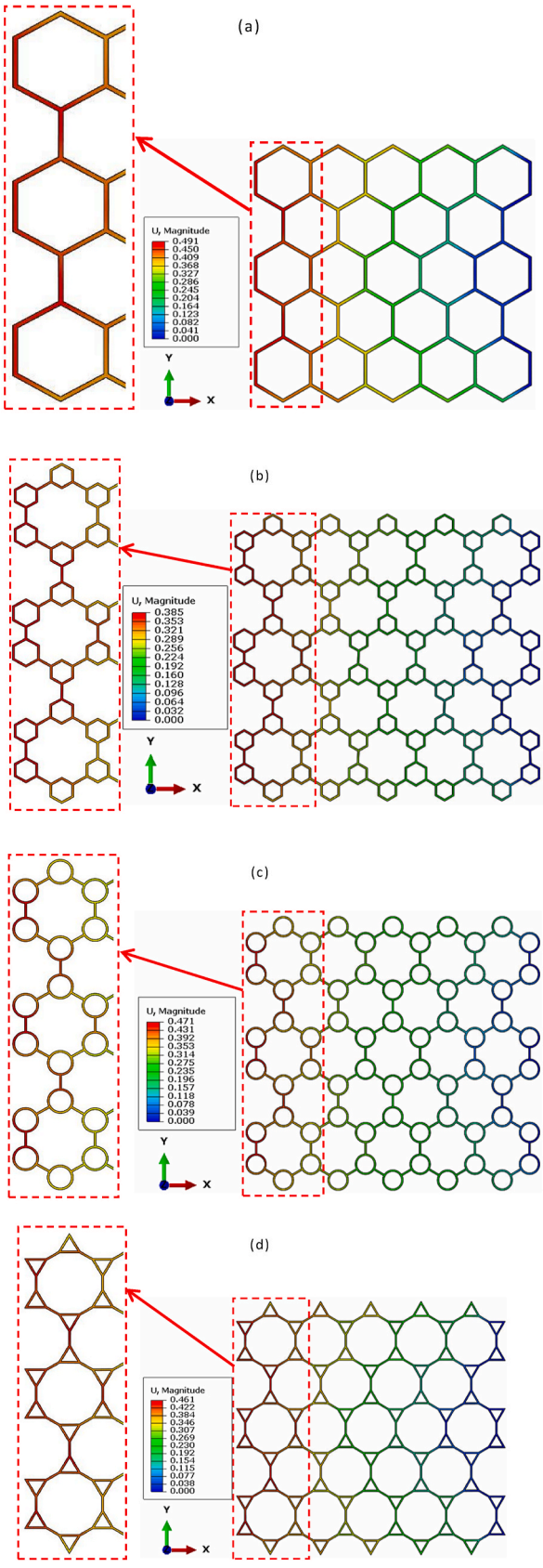


Fig. 13. Numerical results of deflection of the (a) HCR model, (b), (c) and (d) HCH, HCC and HCT models, respectively, when loaded in the *x*-direction.

broad implications for future experimental methods in studies of lattice structures. The observed relationship between lower maximum von Mises stresses and increased load-bearing capacity in the present work suggests the need for new design concepts for lattice structures. There is a need for engineers and researchers to investigate configurations that optimise the directions of load transfer and contact areas in order to improve structural performance of lattice structures. The foregoing is expected to contribute to the growth of design methods for lattice structures.

Experimental results obtained in literature [19,29,30] concur with the numerical models implemented in the current work for hierarchical HC structures, that vertices are areas that experience high stresses for complex hollow structures. Alqassim [29] investigated static and dynamic compression loaded hierarchical HC parts with hexagonal hollow shapes at the vertices. Their research findings suggested that under compression loads, the vertices or nodes tend to fail first. This behaviour of complex hollow shapes has been observed in numerous studies [13–21,28–53] to be purely a function of geometry. The correspondence between the experimental results from Alqassim [53], as well as other literature [57–64] and the numerical results obtained in the current research, is consistent with the expectation that that vertices are prone to high stresses. This consistency gives credibility to the findings of the present work and improves the understanding of the behaviour of hierarchical hollow structures under compression loads. In addition, the observation made in the present work that the behaviour of complicated hollow structures, particularly the capacity for vertices to fail under compression, is purely a function of geometry stresses the significance of geometric considerations in the lattice structural design. This finding is consistent with a broader trend in structural engineering [4,12,40,51,55,56], which highlights the importance of geometric features when determining structural responses. Researchers and engineers working on lattice structures should be aware of the geometric complexities that influence stress distribution and failure mechanisms. The observed sensitivity of vertices to high stresses and failure under compression loads in the present work has direct design implications for lattice structures [51–53]. Engineers as well as designers should think about vertex reinforcement strategies or alternative configurations to reduce stress concentrations in these key areas. The foregoing findings offer useful information for optimising lattice structure designs to improve total structural performance and durability. Understanding how vertices fail under specific loading conditions is critical for risk assessment and safety considerations in general engineering structures as well. This influences decisions about load-carrying capacities, maintenance routines, and safety factors when using lattice structures in multiple kinds of engineering applications, ranging from architecture to aerospace. The recurrent result on the behaviour observed at the vertices of these hollow structures for numerous studies [4,46,51,56] indicates that more research into the mechanics of vertices in complicated hollow structures is required. Future research ought to investigate more geometric configurations, loading conditions, and materials to acquire a better understanding of behaviour of lattices around vertices. This could in turn lead to novel design concepts and methods of structural optimisation.

According to the numerical models generated in the present work, it has been shown that numerically modelled HCH and HCC structures alternate in ranking first with regard to the lowest values of maximum von Mises stresses obtained for the two planar loading directions when compared with the HCR structure, respectively. The highest load bearing capacity in the x-direction was observed for the HCH model, and in the y-direction for the HCC model. The HCH model therefore has the highest load-bearing capacity in the x-direction, whereas the HCC model has the highest load-bearing capacity in the y-direction of the three hierarchical HC structures modelled here. This is a key finding as literature does not give a ranking of such hierarchical structures with various polygon shapes at the vertices with regard to the stress levels. Rather, most research [13–21,31–39] has concentrated on their energy absorption behaviour. The fact that HCH and HCC structures alternatively

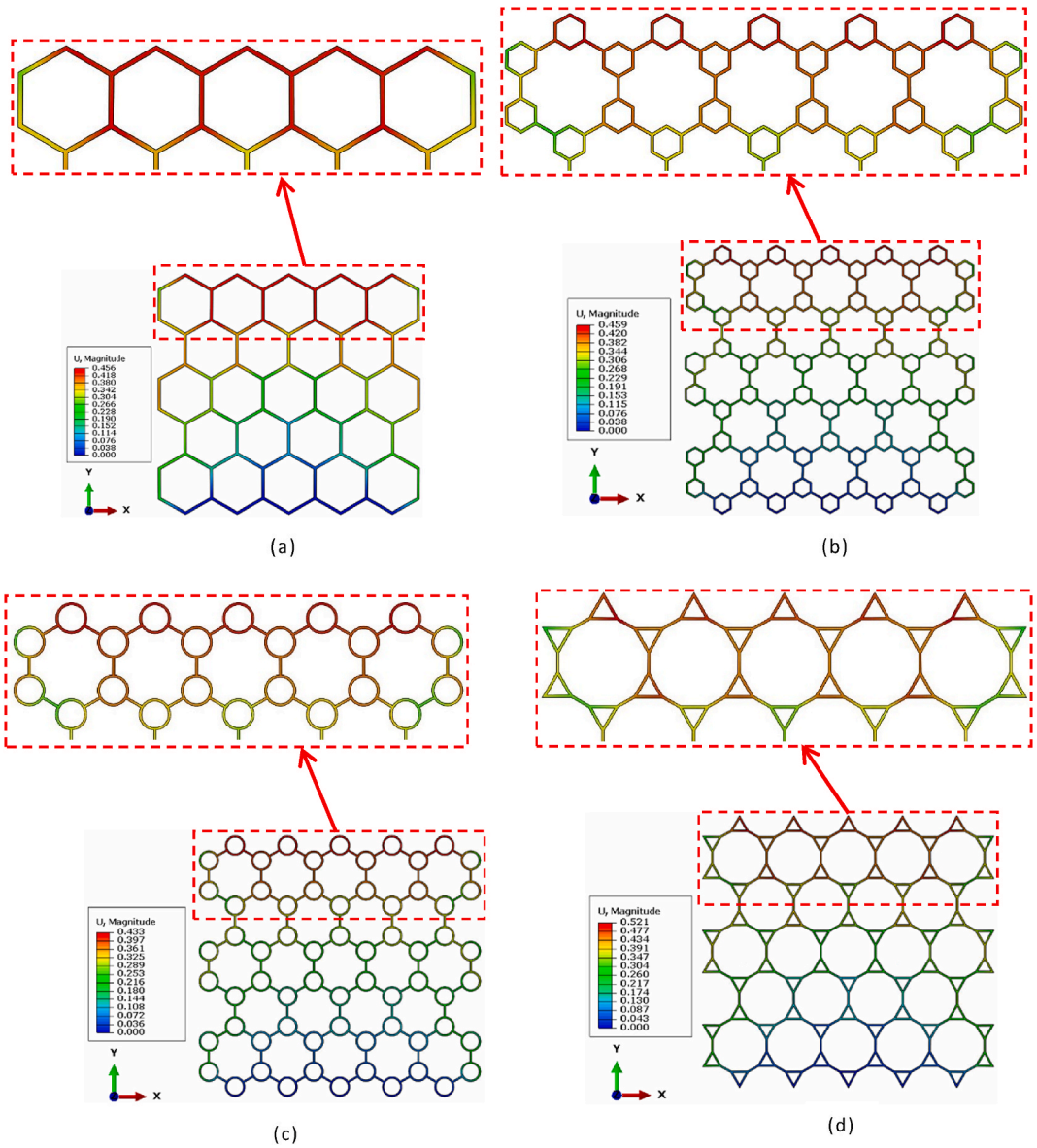


Fig. 14. Numerical results of deflection of the (a) HCR model, (b), (c) and (d) HCH, HCC, and HCT models, respectively, when loaded in the *y*-direction.

score first in different loading directions shows a potential for structural optimisation based on specific loading conditions. Engineers may be able to tailor lattice structures with HCH configurations for improved performance in one loading direction whilst using HCC configurations in another. The research presented here adds to our understanding of how lattice designs can being tailored to improve performance in a variety of real-world applications. The alternating ranking of HCH and HCC structures highlights the relevance of directional strength in lattice structures. Different arrangements could be particularly good at resisting forces applied along different axes. This understanding is especially important for applications where load directionality is vital, such as architectural design, aerospace engineering, or any field in which lattice structures are used for load bearing [4,27,39,51]. The numerical modelling results of the present work produce a fresh feature in literature of ranking HCH and HCC structures based on maximum von Mises stresses. While previous work largely have studied the energy absorption behavior of hierarchical hollow structures [39–44,57–64], this result presents different approach in which lattice structures are graded in terms of stress levels. This understanding can be used to direct future studies into lattice structures and fine-tune design concepts. Generation of hybrid structures that integrate the advantages of both HCH and HCC designs, resulting in improved performance for different loading conditions. This approach is consistent with the tendency in engineering to tailor designs to specific functional requirements [4,27]. The results obtained here thus, offer potential for collaboration with material scientists to investigate how material properties could potentially be optimised for specific lattice configurations. Additionally, interdisciplinary approach could open the path for advances in materials designed particularly for lattice structures [4,27,65].

The numerical models implemented in the present work showed an identical ranking based on corresponding values of deflection, as was observed for the maximum von Mises stresses. The obtained results suggests that the HCH structure is stiffest in the x-direction, while the HCC structure is stiffest in the y-direction. This is also an important finding given the absence of literature on the ranking of hierarchical HC structures with various polygon hollow shapes at the vertices, with respect to deflection. Experimental validation of this ranking forms part of future planned research work. This research sheds light on the stiffness characteristics of hierarchical hollow structures with different polygon hollow shapes at the vertices. Stiffness, and this research

contributes to our current understanding of directional stiffness in lattice structures [39,51], noting that stiffness is an important aspect in determining how structures respond to applied loads. The absence of previous literature on the ranking of hierarchical hollow structures in terms of deflection highlights the originality of this study. While numerous investigations in the field of lattice structures have focused on energy absorption [39–44] or strength [57–64], this finding broadens our understanding to include the stiffness lattice hollow structures. The stiffness rankings for HCH and HCC structures in different directions provide information for engineers and designers looking to optimise lattice structures for specific applications that are stiffness-dependent. Thus, depending on the directional loading assumptions in a particular case, designers may tailor their designs to attain the best stiffness. This has ramifications for industries requiring high directional stiffness, such as aerospace [4,27,53] and robotics [4,27]. Future research is planned to undertake experimental validation of the stiffness ranking determined here. Experimental validation is crucial for assuring the reliability and application of the determined values of stiffness and ranking. Therefore, future research to these ends could assist to develop and improve present design methodologies as well.

3.4. Extending application of the modelling implemented here to design for additive manufacturing

Incorporating the numerical modelling implemented in the present work into the AM design process is promising as well as a challenging strategy for optimising and improving both the design and manufacturing of complicated structures such as hierarchical hollow structures. Numerous key future prospective advantages that must be considered in design for additive manufacturing (DfAM) are as follows:

Numerical modelling allows for tailoring of the design parameters of a structure and/or its manufacturing process. Considering this, the numerical models in the current work could be extended to tailoring their design parameters related to structural integrity, material usage, and manufacturing speed by modelling the behaviour of these structures based on the design limitations in AM technology. Typically, design limitations in AM technology include the maximum size of support structures and the minimum thickness that can be printed for these intricate hollow designs. Additionally, by predicting the behaviour of the numerical models implemented here prior, one could succeed in identifying the most relevant potential challenges or defects in a structural design without the need for manufacturing physical prototypes or parts. Extending the numerical modelling implemented here to the preceding proposed approach reduces prototyping expenses while shortening the manufacturing period. This should be considered a more efficient method to adopt for the design and manufacturing process of complex structures. Numerical modelling is also useful for determining the best material for a given application. In the case of Ti6Al4V(ELI), for instance, the current numerical models could be extended to predict how this material might accurately be used to fabricate geometrical properties such as the complex surfaces. This should include extending the current models to model the response of the material to different mechanical loads based of complex design structures. Alternative numerical modelling packages designed to tackle identical design problems should be implemented to confirm the reliability of the current numerical models. This is performed to verify that design specifications are met before the designs are approved for manufacture. Modelling insights garnered here have the potential of instructing engineers and designers in the key concepts for DfAM and assist in extending the comprehension and expertise in the bioinspired engineering field.

However, there are a number of challenges and factors that must be considered as well in incorporating the numerical models implemented here in DfAM. The level of accuracy of the numerical models used in the present work is critical. To confirm that the data that is incorporated in the DfAM process for complex structures is accurate, these numerical models must be confirmed against physical testing. Complex numerical modelling, in particular the one used in the present work, typically necessitate large computational resources and time. Considerations as to whether such numerical models are likely to be completed in a fair amount of time is necessary. Material properties tend to vary in structures, and the numerical models in the current work are likely to fail to account for this variability, resulting in some inaccuracies in the predictions made regarding the behaviour of complex structures. Though AM allows for greater design flexibility, there are still constraints and limitations relating to the effects of build orientation, and the use of support structures. It is crucial to strike a balance between design flexibility and these constraints. Managing large amounts of data generated by modelling, as well as tracking and controlling changes to the software code could also become a challenge.

Integrating numerical modelling into the AM design process, particularly for complicated structures such as hierarchical honeycomb designs, provides several advantages in terms of optimisation of design, reduction of cost, and increase of performance. However, careful consideration of the accuracy of models, computational resources, and required knowledge is necessary to guarantee the successful implementation of the models. Collaboration with AM and simulation experts is likely to be useful in this effort.

4. Conclusions

Numerical modelling of HCR, HCH, HCC, and HCT structures was conducted to determine the effect of these hierarchical configurations on stresses and deflection. The followings conclusions arise from this work:

- The hierarchical HC structure loaded in the x-direction achieved lower values of maximum von Mises stresses than when loaded in the y-direction. This implies that the mechanical properties of hierarchical lattice structures must be tailored for various structural applications in accordance with the load transfer direction.
- The HCH structure showed the lowest value of maximum of von Mises stress when load was applied in the x-direction. The structure is thus recommended for higher load bearing capacity along the x-direction.
- When loaded in the y-direction, the HCC structure achieved the lowest value of maximum of von Mises stress. Thus, this structure is recommended for load bearing capacity along the y-direction. The outcome contributes to our understanding of how lattice designs could be tailored to improve structural performance. The alternating rankings of HCH and HCC structures underline the importance of directional strength in lattice architectures.
- The HCH structure exhibited the lowest deflection for loading in the x-direction, and is therefore, the stiffest structure for this direction of loading of all the structures analysed here.
- The HCC structure was determined to be the stiffest for y-direction loading because it had the lowest maximum value of deflection in this direction. The study broadens the existing knowledge base and paves the way for subsequent studies into the deflection behaviour of complex lattice structures. The stiffness rankings for HCH and HCC structures in different orientations are useful for engineers and designers who intend to optimise lattice structures for specific uses. Depending on the directional loading assumptions in a particular case, designers can customize their designs to attain the most optimal stiffness.

Future research should aim at production of additive manufactured hierarchical HC parts with geometries similar to the ones studied here for purposes of undertaking experimental validation of the results obtained in the present work. Moreover, future studies should investigate possibilities of improving the load-bearing capacity of the hierarchical HC structures studied here, though topology and shape optimisation of the highly stressed regions identified in this study.

CRediT authorship contribution statement

M.I. Chibinyani: Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **T.C. Dzogbewu:** Conceptualization, Funding acquisition, Supervision, Validation, Writing – review & editing. **M. Maringa:** Conceptualization, Funding acquisition, Methodology, Supervision, Visualization, Writing – review & editing. **A.M. Muiruri:** Conceptualization, Methodology, Validation, Visualization, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

Funding: This work is based on research supported by the South African Research Chairs Initiative of the Department of Science and Technology, and the National Research Foundation of South Africa (Grant No. 97994), and the Collaborative Program in Additive Manufacturing (Contract No. CSIR-NLC-CPAM-21-MOA-CUT-03).

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